

**Radiation-hydrodynamics  
from Mpc to sub-pc scales  
with RAMSES-RT**

**Joki Rosdahl (CRAL)**

With

Aubert, Blaizot, Bieri, Biernacki, Commercon, Costa, Courty, Dubois,  
Geen, Katz, Kimm, Nickerson, Perret, Schaye, Stranex, Teyssier, Trebitsch

**ASTROSIM summer school, July 7th, 2017**

# What is RAMSES-RT?

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## Multi-purpose radiation-hydrodynamics

Rosdahl et al. (2013)

Rosdahl & Teyssier (2015)

- Part of the cosmological code RAMSES (Teyssier '01)
- Publicly available ([www.bitbucket.org/rteyssie/ramses](http://www.bitbucket.org/rteyssie/ramses))
- Emission of photons from e.g. stars, AGN, gas
- Transport of photons through the 3D volume, on-the-fly with hydro, and in adaptive mesh refinement
- Hydro-coupled absorption and scattering by gas and dust
  - Photoionisation and heating of H, He, H<sub>2</sub> (and eager to add more!)
  - Radiation pressure, i.e. momentum transfer from photons to gas
  - Multi-scattering on dust

# How do ionising photons interact with primordial gas?

photoionisation, heating, pressure, recombinations



**What is RAMSES-RT for?**

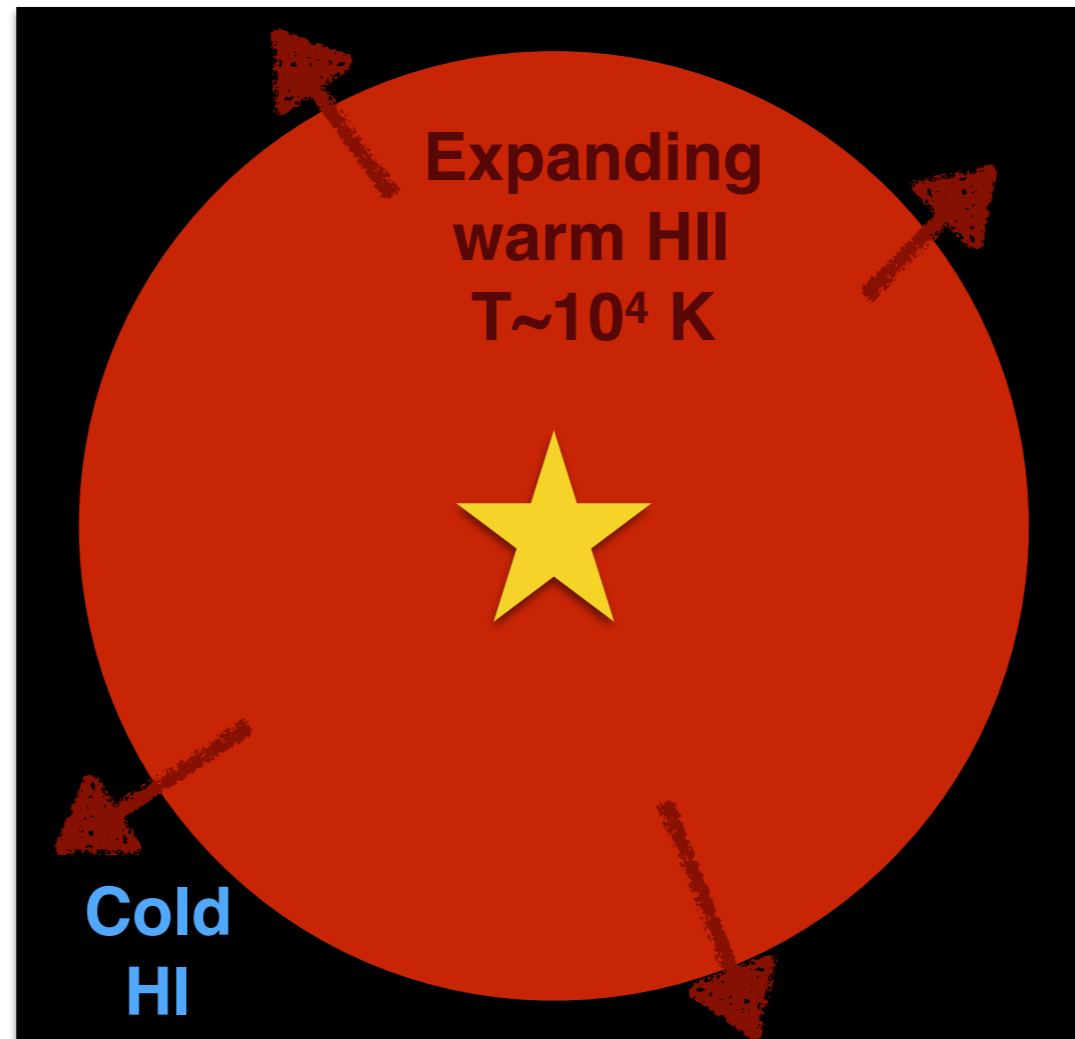
# What is RAMSES-RT for?

- **Observable properties of gas in and around galaxies**  
Rosdahl+12, Katz+17
- **Stellar/AGN radiation feedback** Rosdahl+15, Geen+15-16-17,  
Bieri+16, Gavagnin+17, Costa+17
- **Ionising radiation escape from galaxies** Kimm+14, Kimm+17,  
Trebtsch+17, Katz+17
- **Large-scale reionisation** (recently feasible with variable light speed)
- **H<sub>2</sub> formation and destruction** Butler+17
- **Protostar formation**

## ...but not for...

- **'Line' radiative transfer and PDR diagnostics** ☞ Monte Carlo
  - **Situations where strong shadows are important**
- ➡ **More about dynamical effects of radiation than diagnostics**

# Photoionisation heating



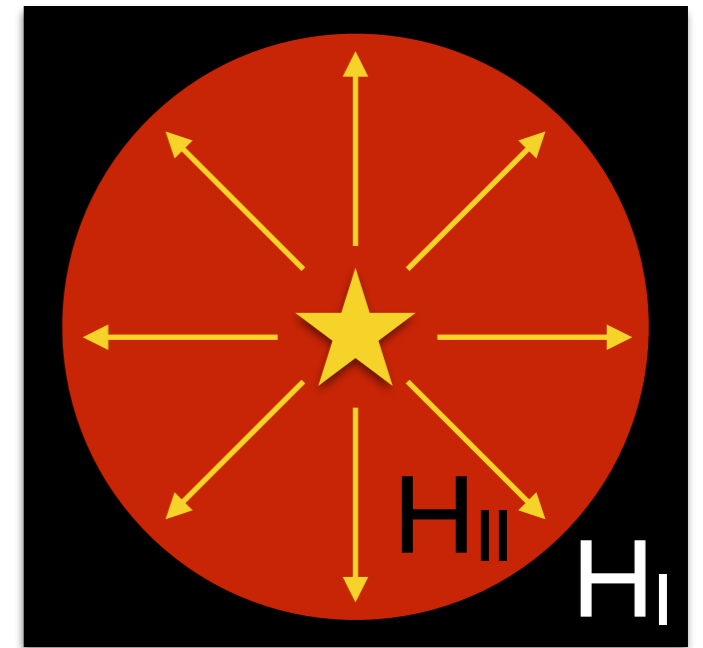
# Radiation pressure

Momentum absorption

Stellar UV luminosity

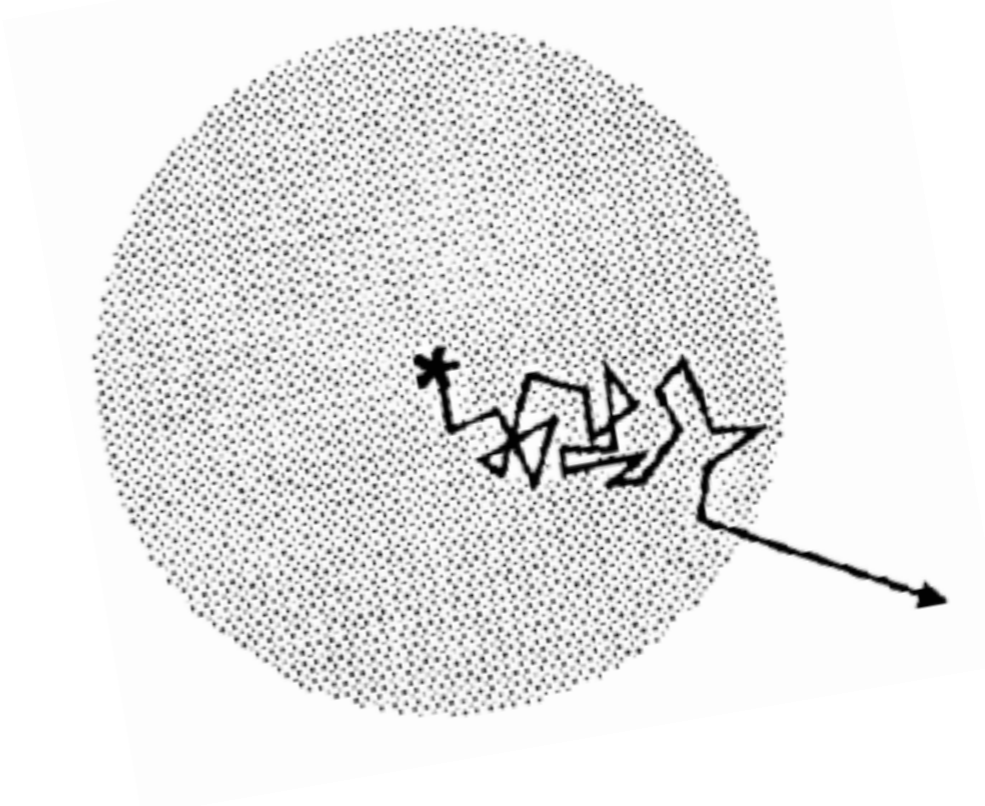
UV optical depth

$$\dot{p}_{\text{rad}} = \frac{L_{\text{UV}}}{c} (1 - e^{-\tau_{\text{UV}}}) \approx \frac{L_{\text{UV}}}{c}$$



IR radiation pressure on dust can be stronger because of **multi-scattering** pressure boost

$$\dot{p}_{\text{rad}} = \frac{L_{\text{Opt}}}{c} \tau_{\text{IR}}$$

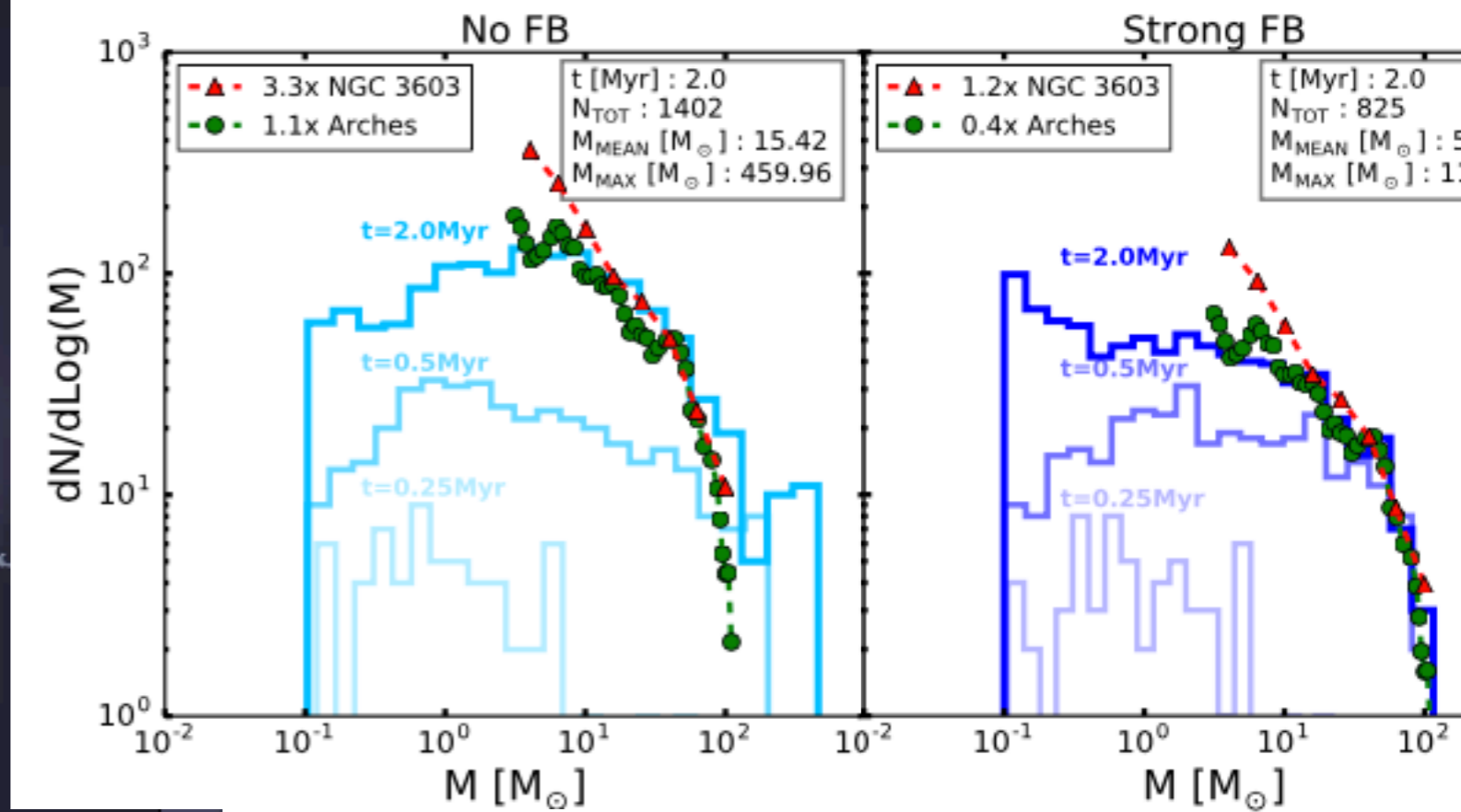
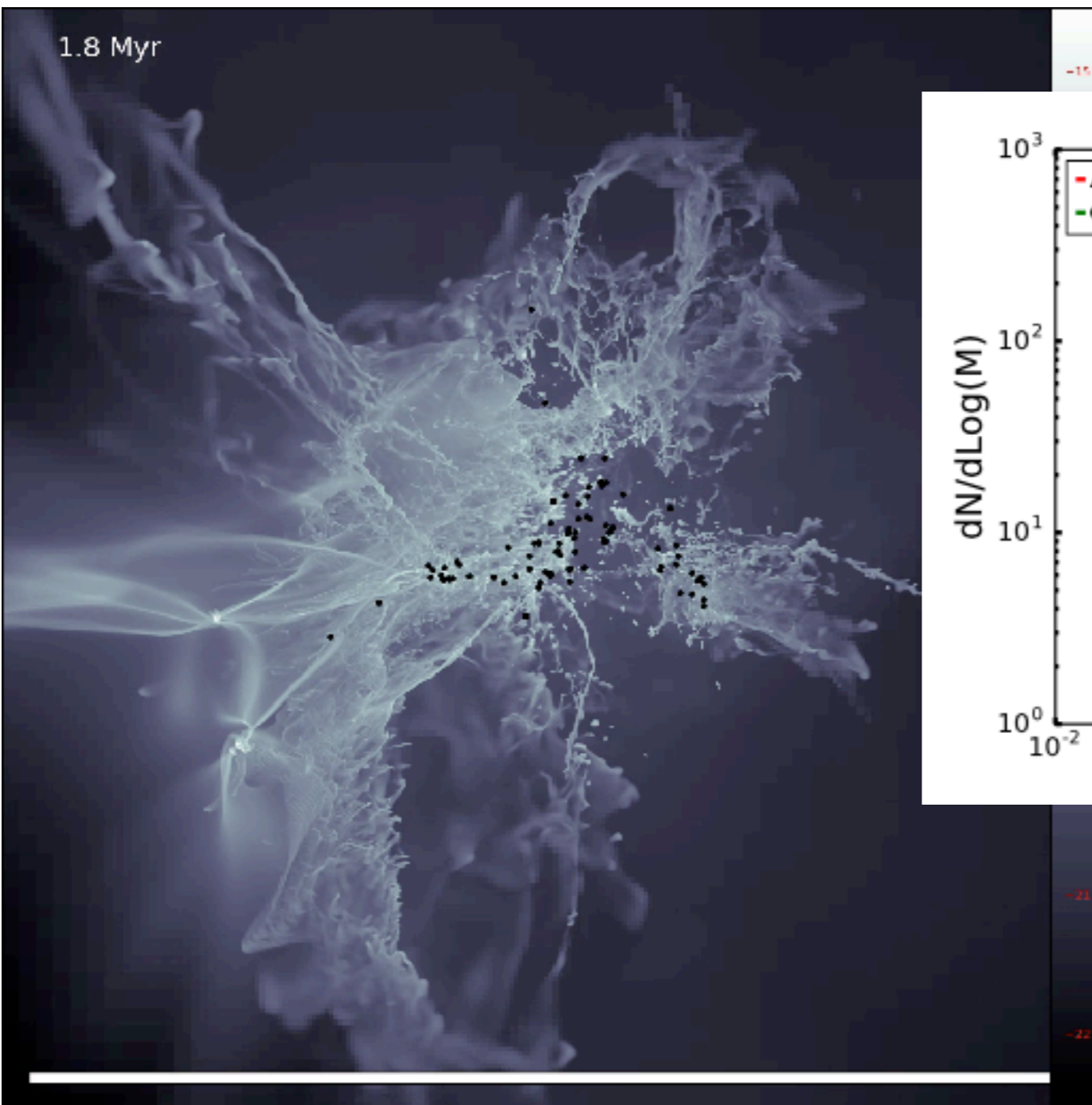




# Photoionisation feedback in molecular clouds with RAMSES-RT

Gavagnin et al. (2017)

Effect of ionising radiation on emerging stellar population and runaway stars



From Gavagnin et al. (2017)

# Overview

Challenges in numerical radiative transfer

Main features of RAMSES-RT

M1 moment radiative transfer

Reduced and variable speed of light

Coupling to AMR hydrodynamics

Radiation pressure and multi-scattering

Tests

Science with RAMSES-RT

# The radiative transfer equation

main challenges in numerical approaches

$$\frac{1}{c} \frac{\partial I_\nu}{\partial t} + \mathbf{n} \cdot \nabla I_\nu = -\kappa_\nu I_\nu + \eta_\nu$$

$I_\nu(\mathbf{x}, \mathbf{n}, t)$  intensity

$\kappa_\nu(\mathbf{x}, \mathbf{n}, t)$  absorption

$\eta_\nu(\mathbf{x}, \mathbf{n}, t)$  source function



To solve this numerically, we need to overcome two main problems:

# The radiative transfer equation

main challenges in numerical approaches

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Frequency ↗  
Location ↗  
Angle ↑  
Time ↘

To solve this numerically, we need to overcome two main problems:

I. There are seven dimensions! Hydrodynamics have only four!

# The radiative transfer equation

main challenges in numerical approaches

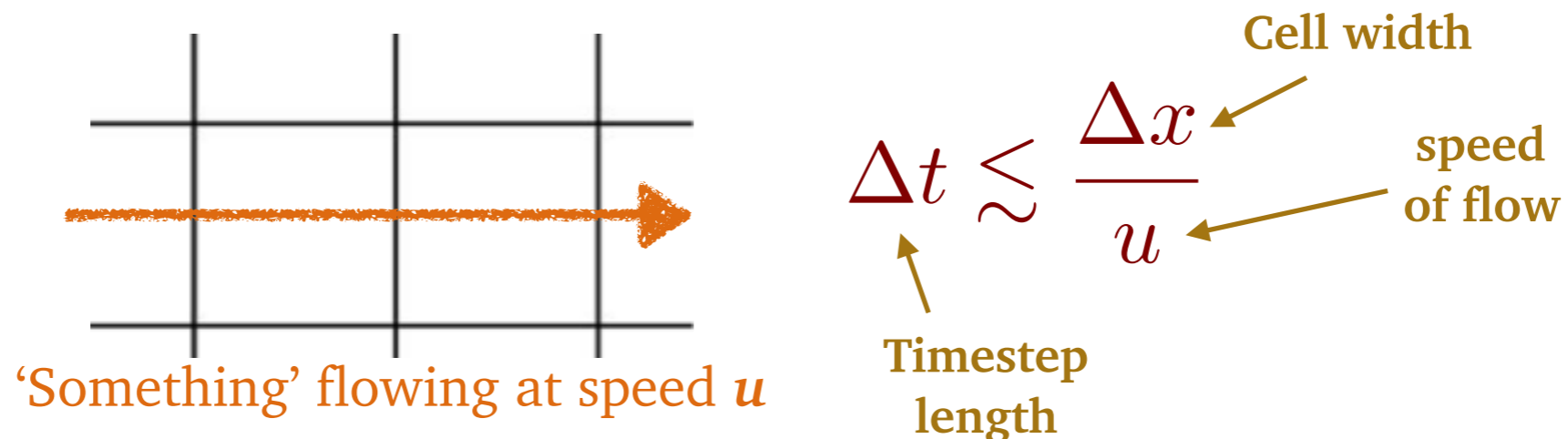
$$\frac{1}{c} \frac{\partial I_\nu}{\partial t} + \mathbf{n} \cdot \nabla I_\nu = -\kappa_\nu I_\nu + \eta_\nu$$

$I_\nu(\mathbf{x}, \mathbf{n}, t)$  intensity  
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 $\eta_\nu(\mathbf{x}, \mathbf{n}, t)$  source function

Frequency ↗  
Location ↗  
Angle ↑  
Time ↘

To solve this numerically, we need to overcome two main problems:

- I. There are seven dimensions! Hydrodynamics have only four!
- II. The timescale is  $\propto u^{-1}$ , where  $u$  is speed, and  $u_{\text{light}} \sim 1000 u_{\text{gas}}$ , so  $\sim$  thousand RT steps per hydro step!!



# The radiative transfer equation

main challenges in numerical approaches

$$\frac{1}{c} \frac{\partial I_\nu}{\partial t} + \mathbf{n} \cdot \nabla I_\nu = -\kappa_\nu I_\nu + \eta_\nu$$

$I_\nu(\mathbf{x}, \mathbf{n}, t)$  intensity

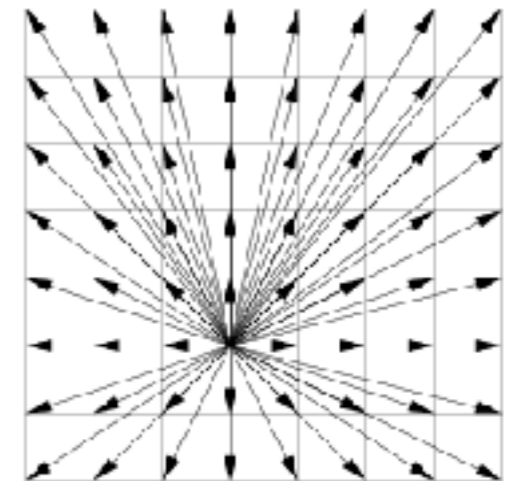
$\kappa_\nu(\mathbf{x}, \mathbf{n}, t)$  absorption

$\eta_\nu(\mathbf{x}, \mathbf{n}, t)$  source function

## Two common strategies for radiative transfer:

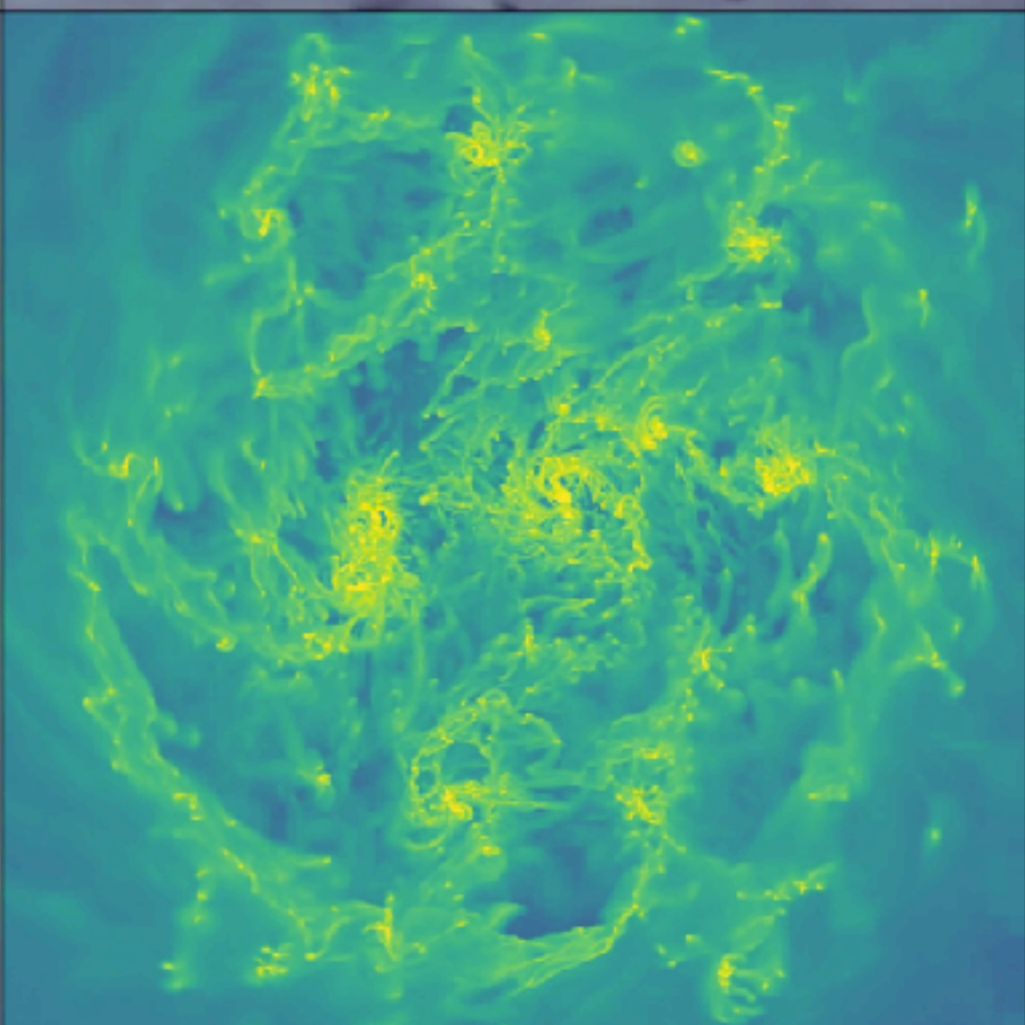
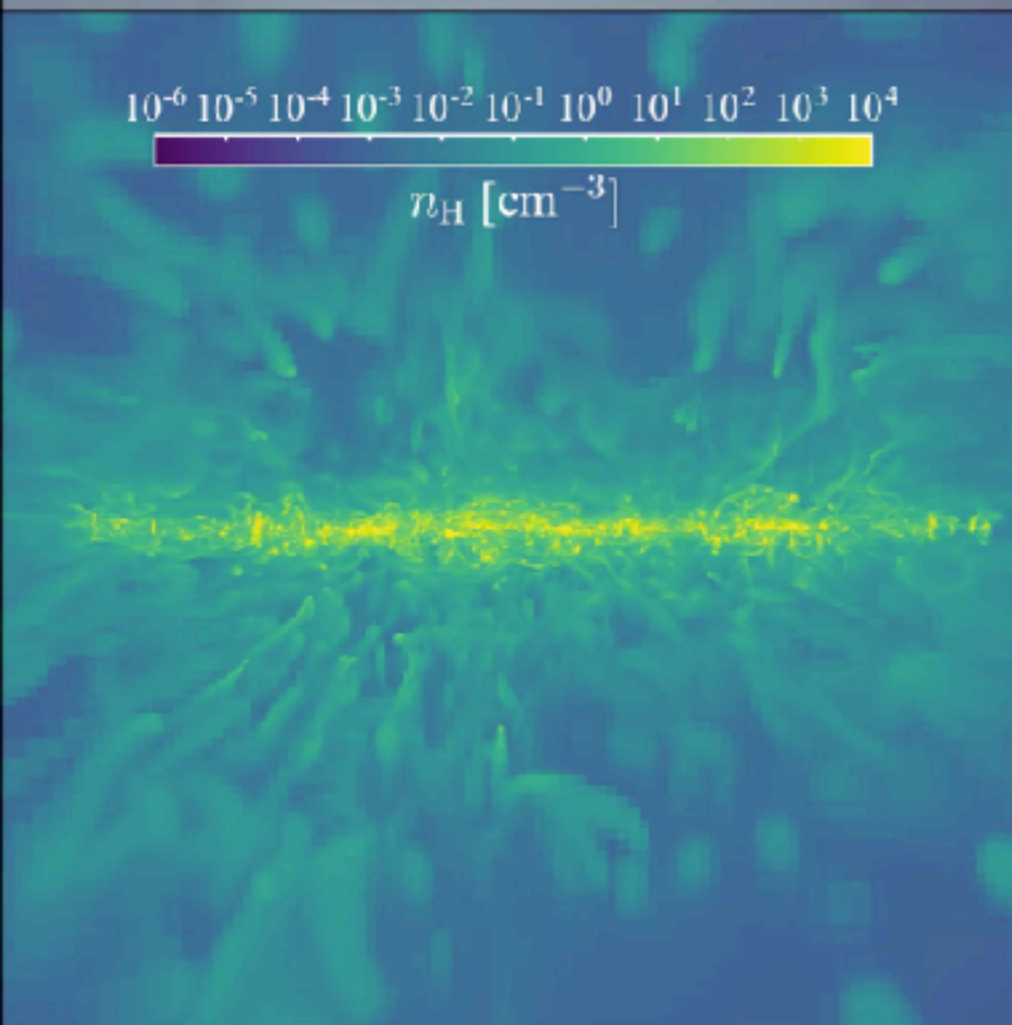
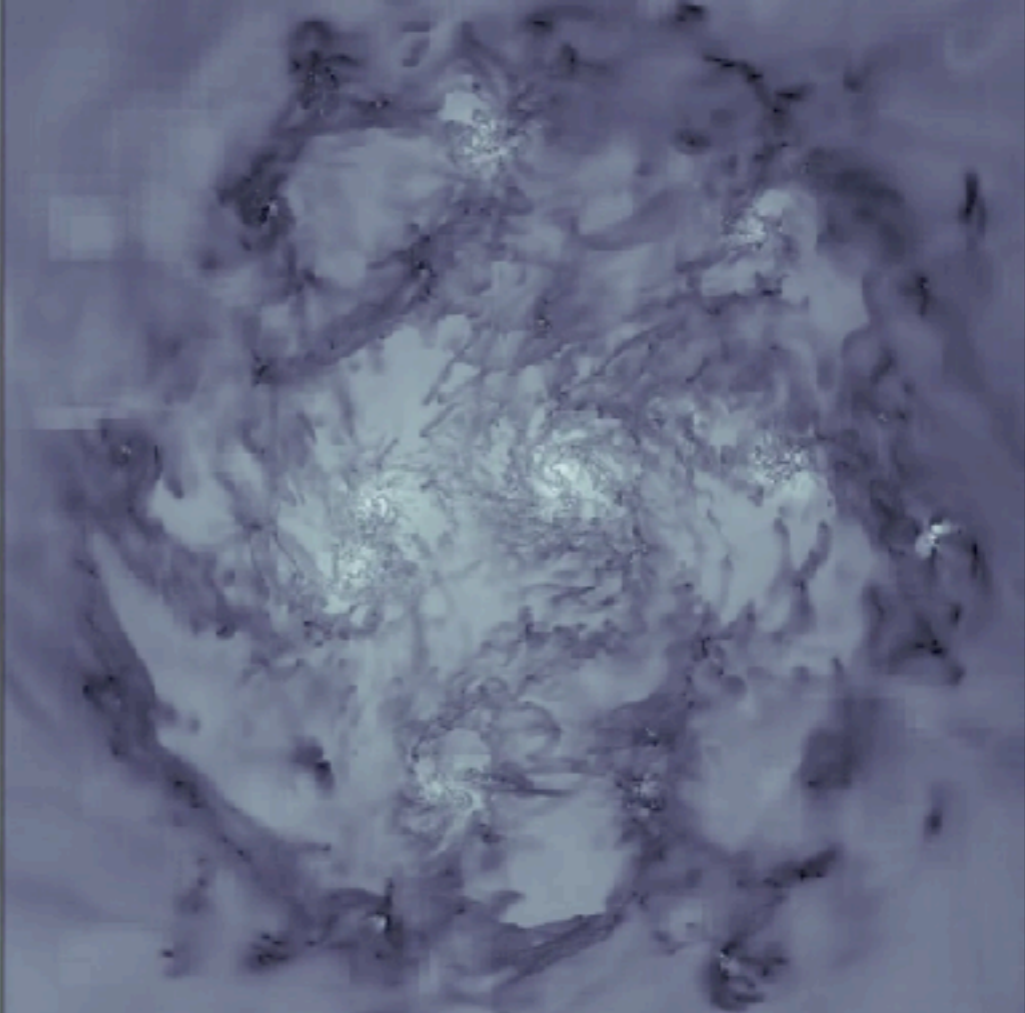
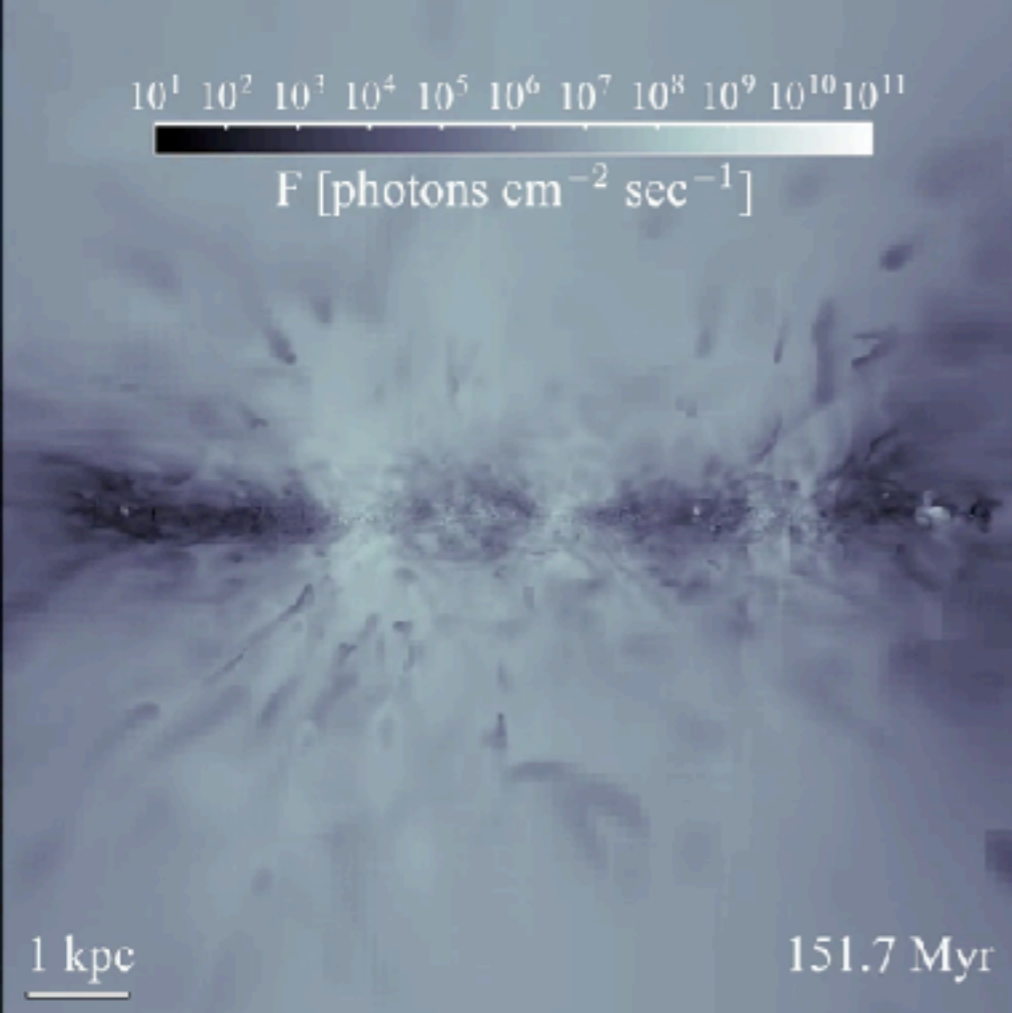
I. *Ray tracing methods*: Cast a finite number of rays from a finite number of sources

- Simple and intuitive
- ...but efficiently covering the volume can be tricky
- ...and load scales with number of sources/rays



II. *Moment methods*: Convert the RT equation into a system of conservation laws that describe a *field* of radiation

- Not so intuitive, and *not rays*
- ...but fits easily with a hydrodynamical solver for RHD
- ...naturally takes advantage of AMR and parallelization
- ...no problem with covering the volume
- ...no limit to number of radiation sources



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## Main features of RAMSES-RT

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# Moments of the RT equation

to get rid of the angular dimension

$$\frac{1}{c} \frac{\partial I_\nu}{\partial t} + \mathbf{n} \cdot \nabla I_\nu = -\kappa_\nu I_\nu + \eta_\nu$$

$I_\nu(\mathbf{x}, \mathbf{n}, t)$  intensity

$\kappa_\nu(\mathbf{x}, \mathbf{n}, t)$  absorption

$\eta_\nu(\mathbf{x}, \mathbf{n}, t)$  source function

Zeroth moment:  $\oint f(\mathbf{n}) d\Omega$

$$\frac{1}{c} \frac{\partial}{\partial t} \oint I_\nu d\Omega + \nabla \cdot \oint \mathbf{n} I_\nu d\Omega = -\kappa_\nu \oint I_\nu d\Omega + \eta_\nu \oint d\Omega$$

First moment:  $\oint \mathbf{n} f(\mathbf{n}) d\Omega$

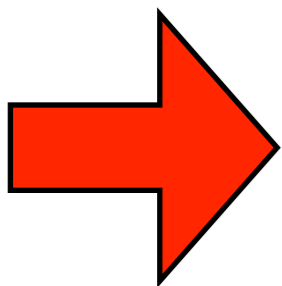
$$\frac{1}{c} \frac{\partial}{\partial t} \oint \mathbf{n} I_\nu d\Omega + \nabla \cdot \oint \mathbf{n} \otimes \mathbf{n} I_\nu d\Omega = -\kappa_\nu \oint \mathbf{n} I_\nu d\Omega$$

These equations contain the first three moments of the intensity:

$$E_\nu = \frac{1}{c} \oint I_\nu d\Omega \quad (\text{energy per volume and frequency})$$

$$\mathbf{f}_\nu = \oint \mathbf{n} I_\nu d\Omega \quad (\text{energy flux per area and time and frequency})$$

$$\mathbb{P}_\nu = \frac{1}{c} \oint \mathbf{n} \otimes \mathbf{n} I_\nu d\Omega \quad (\text{force per area and frequency})$$



$$\frac{\partial E_\nu}{\partial t} + \nabla \cdot \mathbf{f}_\nu = -\kappa_\nu c E_\nu + S_\nu$$
$$\frac{\partial \mathbf{f}_\nu}{\partial t} + c^2 \nabla \cdot \mathbb{P}_\nu = -\kappa_\nu c \mathbf{f}_\nu$$

# Moment RT equations

$$\frac{\partial E_\nu}{\partial t} + \nabla \cdot \mathbf{f}_\nu = -\kappa_\nu c E_\nu + S_\nu$$

$$\frac{\partial \mathbf{f}_\nu}{\partial t} + c^2 \nabla \cdot \mathbb{P}_\nu = -\kappa_\nu c \mathbf{f}_\nu$$

$$E_\nu = \frac{1}{c} \oint I_\nu d\Omega \quad (\text{energy per volume and frequency})$$

$$\mathbf{f}_\nu = \oint \mathbf{n} I_\nu d\Omega \quad (\text{energy flux per area and time and frequency})$$

$$\mathbb{P}_\nu = \frac{1}{c} \oint \mathbf{n} \otimes \mathbf{n} I_\nu d\Omega \quad (\text{force per area and frequency})$$

With ionizing radiation, it makes more sense to keep track of *photon number density* than energy density

$$\frac{\partial N_\nu}{\partial t} + \nabla \cdot \mathbf{F}_\nu = - \sum_j^{\text{HI,HeI,HeII}} n_j c \sigma_{\nu j} N_\nu + \dot{N}_\nu^* + \dot{N}_\nu^{rec}$$

$$\frac{\partial \mathbf{F}_\nu}{\partial t} + c^2 \nabla \cdot \mathbb{P}_\nu = - \sum_j^{\text{HI,HeI,HeII}} n_j c \sigma_{\nu j} \mathbf{F}_\nu$$

$N(\mathbf{x}, t)$  photon density

$\mathbf{F}(\mathbf{x}, t)$  photon number flux

$\mathbb{P}(\mathbf{x}, t)$  photon 'pressure'

# Frequency binning

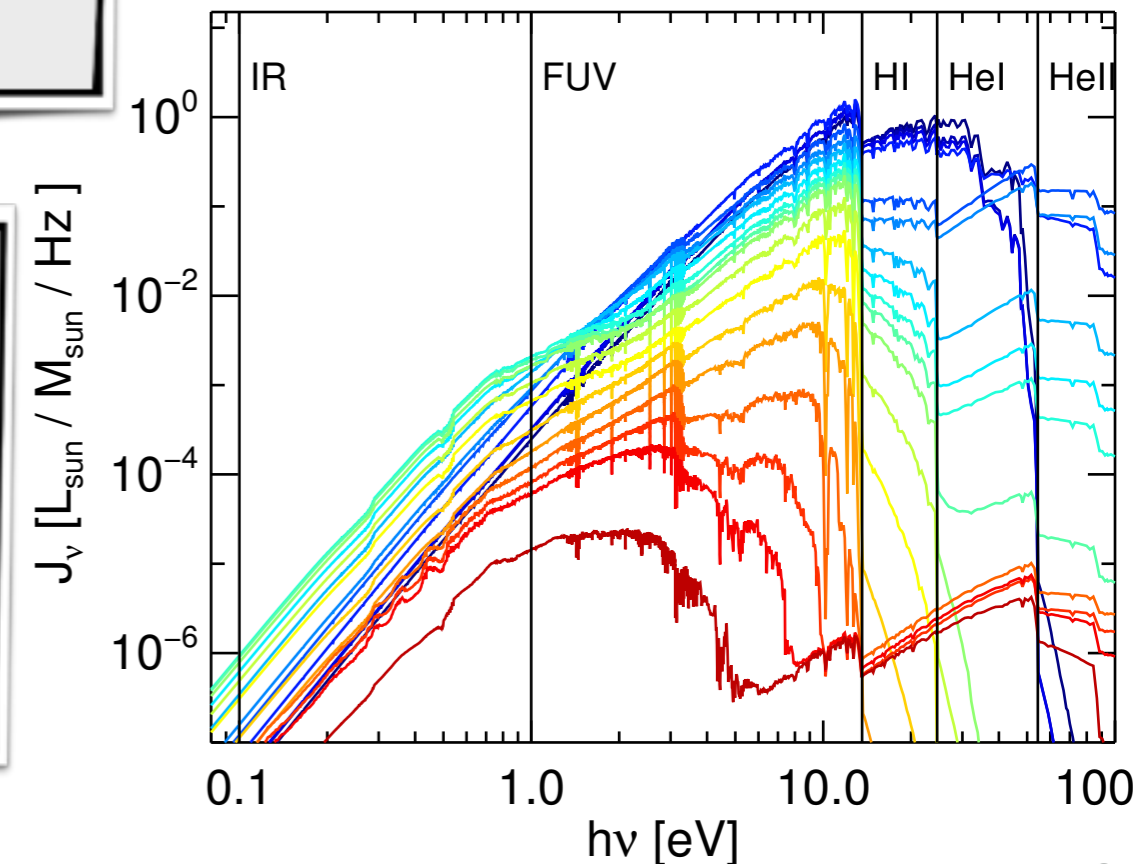
We separate the RT equations into  $M$  sets, or photon groups, that discretise the frequency continuum

➔  $M$  (a handful of) separate radiation fields, one per frequency bin, with average photon energies and cross sections

$$\frac{\partial N_\nu}{\partial t} + \nabla \cdot \mathbf{F}_\nu = - \sum_j^{\text{HI, HeI, HeII}} n_j c \sigma_{\nu j} N_\nu + \dot{N}_\nu^* + \dot{N}_\nu^{rec}$$

$$\frac{\partial \mathbf{F}_\nu}{\partial t} + c^2 \nabla \cdot \mathbb{P}_\nu = - \sum_j^{\text{HI, HeI, HeII}} n_j c \sigma_{\nu j} \mathbf{F}_\nu$$

A SED of stellar populations is used to characterise frequencies and energies of the photon groups



$$\frac{\partial N_i}{\partial t} + \nabla \cdot \mathbf{F}_i = - \sum_j^{\text{HI, HeI, HeII}} n_j c \sigma_{ij} N_i + \dot{N}_i^* + \dot{N}_i^{rec}$$

$$\frac{\partial \mathbf{F}_i}{\partial t} + c^2 \nabla \cdot \mathbb{P}_i = - \sum_j^{\text{HI, HeI, HeII}} n_j c \sigma_{ij} \mathbf{F}_i$$

# Radiation variables, stored in each cell

one set per photon group (neglecting the i-index)

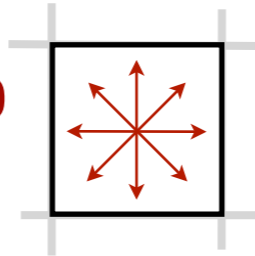
$$\begin{aligned} \frac{\partial N}{\partial t} + \nabla \cdot \mathbf{F} &= - \sum_j^{\text{HI, HeI, HeII}} n_j \sigma_j c N + \dot{N}^* + \dot{N}^{rec} \\ \frac{\partial \mathbf{F}}{\partial t} + c^2 \nabla \cdot \mathbb{P} &= - \sum_j^{\text{HI, HeI, HeII}} n_j \sigma_j c \mathbf{F} \end{aligned}$$

$N(\mathbf{x})$  **photon density**

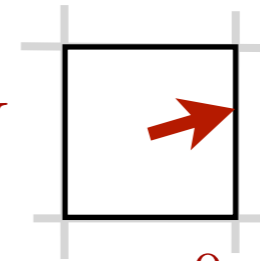
$\mathbf{F}(\mathbf{x}) = (F_x, F_y, F_z)$   
**photon flux**

**Describe isotropic plus directed radiation in each volume element (cell)**

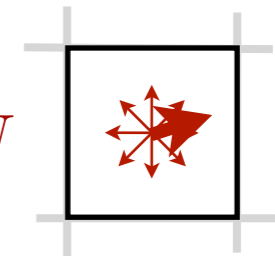
Diffusion limit (isotropic):  $\mathbf{F} = \mathbf{0}$



Transport limit (directed):  $|\mathbf{F}| = cN$



$0 < |\mathbf{F}| < cN$



A combination:

The 'directionality' can be described by the **reduced flux**:  $f \equiv \frac{|\mathbf{F}|}{cN}, \quad 0 \leq f \leq 1$

We almost have usable expressions - we 'only' need to close the equations

# Closing the moment RT equations

$$\frac{\partial N}{\partial t} + \nabla \cdot \mathbf{F} = - \sum_{j \text{ HI, HeI, HeII}} n_j \sigma_j c N + \dot{N}^* + \dot{N}^{rec}$$

$$\frac{\partial \mathbf{F}}{\partial t} + c^2 \nabla \cdot \mathbb{P} = - \sum_{j \text{ HI, HeI, HeII}} n_j \sigma_j c \mathbf{F}$$

$$\mathbb{P} = \mathbb{D}N \quad \text{Eddington tensor}$$

$N(\mathbf{x}, t)$  photon density  
 $\mathbf{F}(\mathbf{x}, t)$  photon flux  
 $\mathbb{P}(\mathbf{x}, t)$  photon 'pressure'

## Three closures:

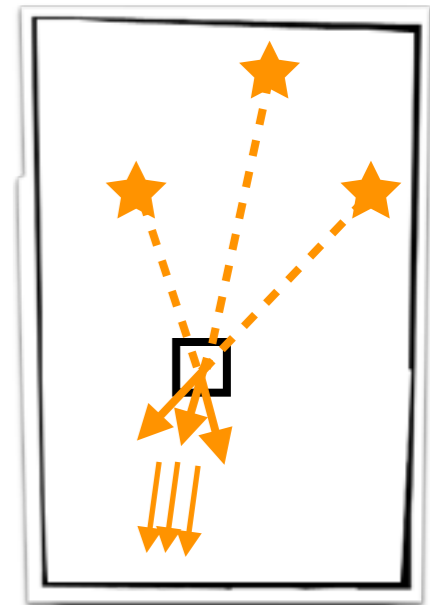
*I: Flux limit diffusion: Radiation flows in the direction of less radiation*

*Good description in the optically thick limit*

*II: (OT)VET: (Optically thin) variable Eddington tensor:*

*The tensor (direction of flow) is made from the sum of all sources*

*Nonlocal expression, so computationally challenging.*



*III: M1 closure (Levermore 1984): The tensor is composed out of the **local** quantities  $N$  and  $\mathbf{F}$  only*

# M1 closure

$$\frac{\partial N}{\partial t} + \nabla \cdot \mathbf{F} = - \sum_j^{\text{HI,HeI,HeII}} n_j \sigma_j c N + \dot{N}^* + \dot{N}^{rec}$$

$$\frac{\partial \mathbf{F}}{\partial t} + c^2 \nabla \cdot \mathbb{P} = - \sum_j^{\text{HI,HeI,HeII}} n_j \sigma_j c \mathbf{F}$$


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$$\mathbb{P} = \mathbb{D} N$$

An expression for  $\mathbb{D}$  must **conserve photons** and **preserve flux**.  
Should also be **local** (i.e. derived only from quantities within the cell).

**Levermore 1984:**

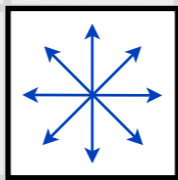
$$\mathbb{D} = \frac{1 - \chi}{2} \mathbf{I} + \frac{3\chi - 1}{2} \frac{\mathbf{F} \otimes \mathbf{F}}{|\mathbf{F}|^2}$$

$$\chi = \frac{3 + 4f^2}{5 + 2\sqrt{4 - 3f^2}}$$

$$f \equiv \frac{|\mathbf{F}|}{cN}$$

## Examples

**Pure diffusion:**



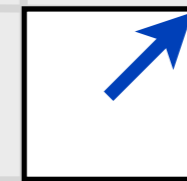
$$\mathbf{F} = \mathbf{0}$$

$$f = 0$$

$$\chi = 1/3$$

$$\Rightarrow \mathbb{P} = \begin{bmatrix} N/3 & 0 & 0 \\ 0 & N/3 & 0 \\ 0 & 0 & N/3 \end{bmatrix}$$

**Pure transport:**



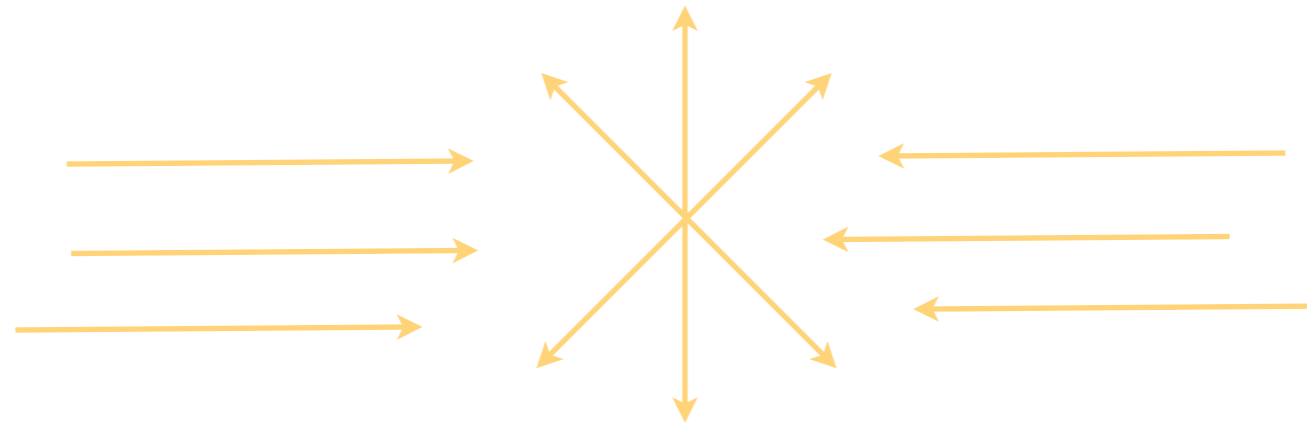
$$\mathbf{F} = cN \left( \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \right)$$

$$f = 1$$

$$\chi = 1$$

$$\Rightarrow \mathbb{P} = \begin{bmatrix} N/2 & N/2 & 0 \\ N/2 & N/2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

# M1 closure - a warning



The loss of the angular dimension combined with the locality of M1 results in peculiar radiation propagation

We sacrifice the collision-less nature of photons

$$\frac{\partial N}{\partial t} + \nabla \cdot \mathbf{F} = - \sum_{j}^{\text{HI, HeI, HeII}} n_j \sigma_j c N + \dot{N}^* + \dot{N}^{rec}$$

$$\frac{\partial \mathbf{F}}{\partial t} + c^2 \nabla \cdot \mathbb{P} = - \sum_{j}^{\text{HI, HeI, HeII}} n_j \sigma_j c \mathbf{F}$$


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$$\mathbb{P} = \mathbb{D}N$$

**Solving the M1 moment RT equations on a uniform grid**



# Solving the RT moment equations on a grid

**Operator splitting:** separate into **3 steps** that can be solved in order over one discrete timestep at a time:

$$t^n \rightarrow t^{n+1} = t^n + \Delta t$$

**3 steps, at each  $\Delta t$ :**

I. Photon transport  
between adjacent cells

II. Injection into cells

III. Thermochemistry in every cell

$$\frac{\partial N}{\partial t} + \nabla \cdot \mathbf{F} = - \sum_{j \in \text{HI, HeI, HeII}} n_j \sigma_j c N + \dot{N}^* + \dot{N}^{rec}$$

$$\frac{\partial \mathbf{F}}{\partial t} + c^2 \nabla \cdot \mathbb{P} = - \sum_{j \in \text{HI, HeI, HeII}} n_j \sigma_j c \mathbf{F}$$


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$$\mathbb{P} = \mathbb{D}N$$

$$\frac{\partial \varepsilon}{\partial t} = \Lambda(\rho, \varepsilon, n_j, N_i)$$

# Photon transport

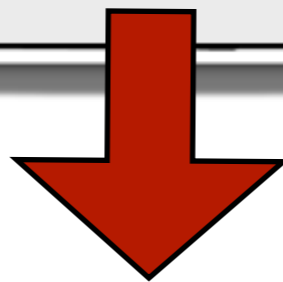
$$\frac{\partial N}{\partial t} + \nabla \cdot \mathbf{F} = - \sum_j^{\text{HI, HeI, HeII}} n_j \sigma_j c N + \dot{N}^* + \dot{N}^{rec}$$

$$\frac{\partial \mathbf{F}}{\partial t} + c^2 \nabla \cdot \mathbb{P} = - \sum_j^{\text{HI, HeI, HeII}} n_j \sigma_j c \mathbf{F}$$


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$$\mathbb{P} = \mathbb{D}N$$

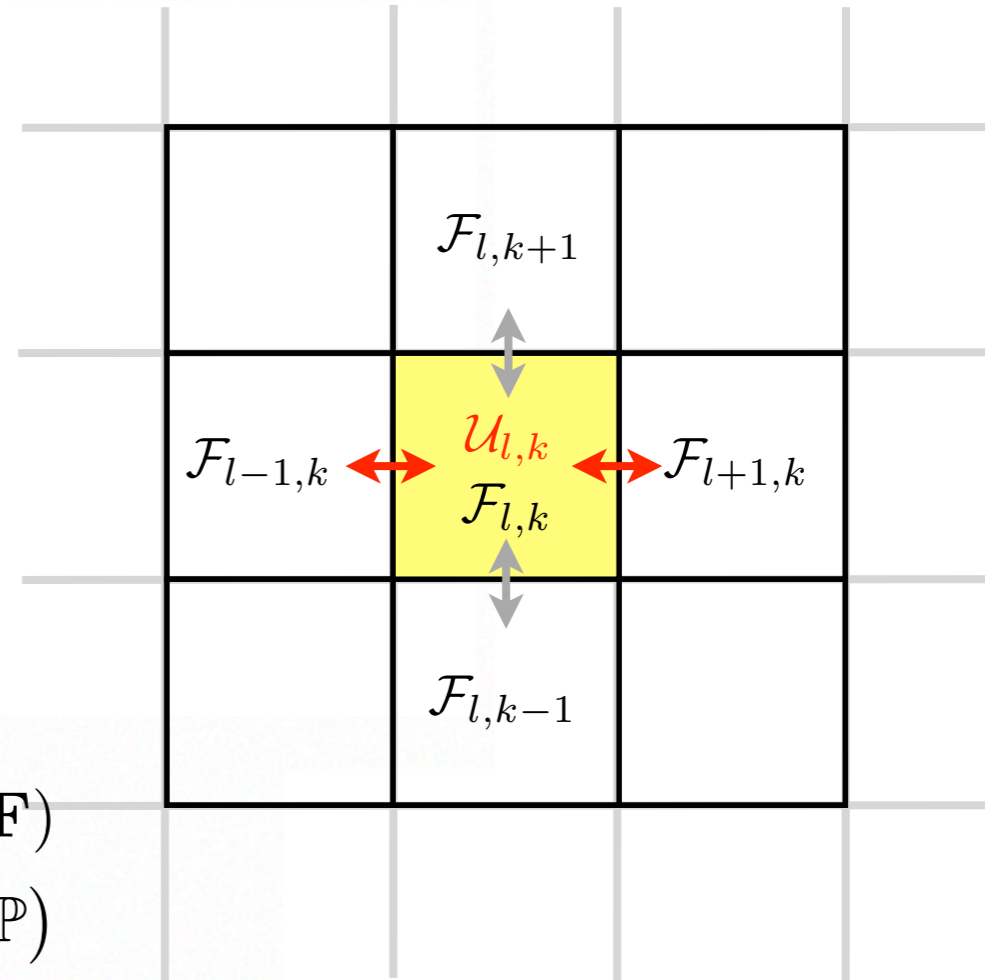
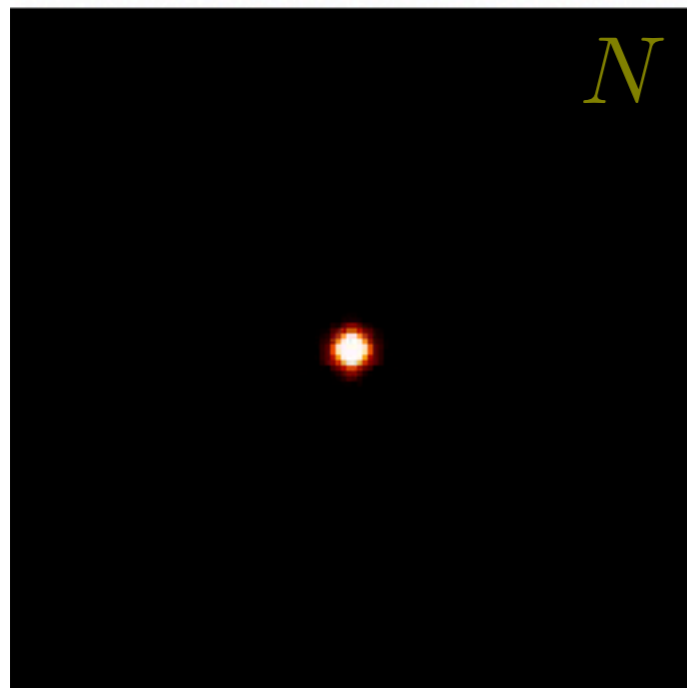
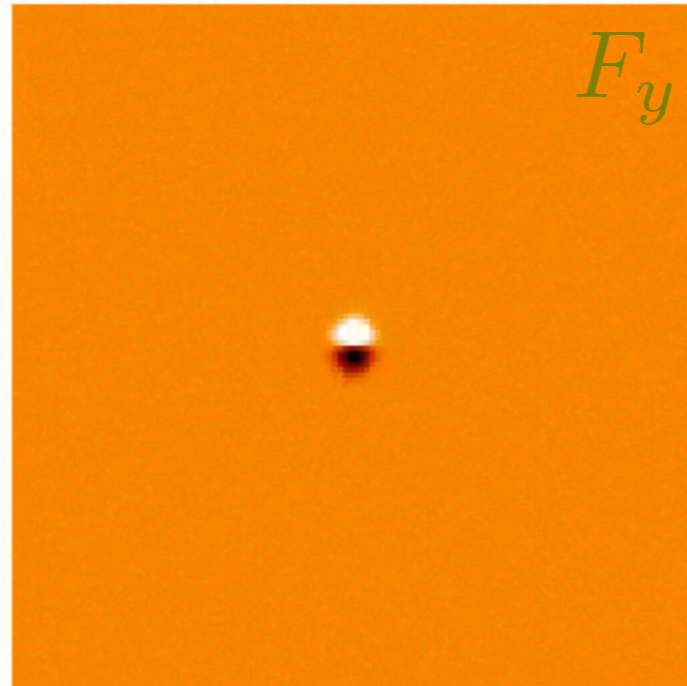
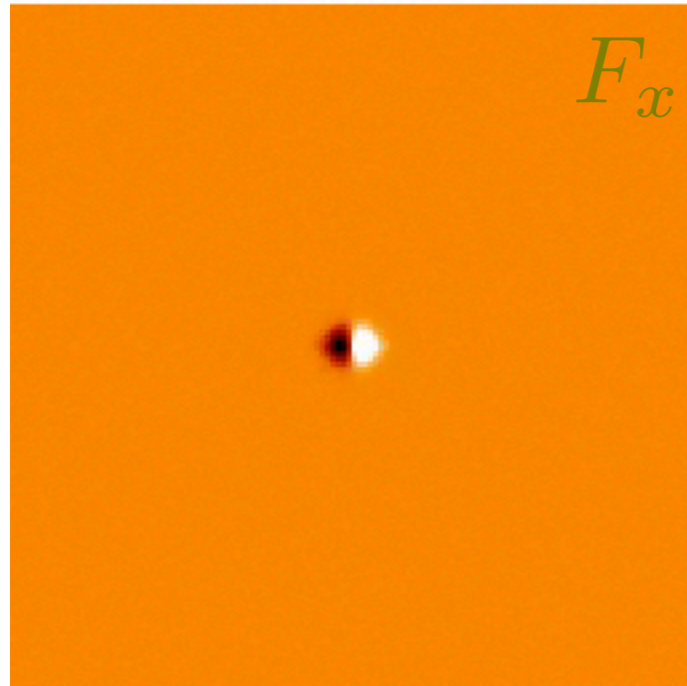
$$\frac{\partial \varepsilon}{\partial t} = \Lambda(\rho, \varepsilon, n_j, N_i)$$



$$\frac{\partial N}{\partial t} + \nabla \cdot \mathbf{F} = 0,$$

$$\frac{\partial \mathbf{F}}{\partial t} + c^2 \nabla \cdot \mathbb{P} = 0,$$

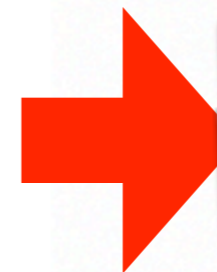
# Photon transport



$$\mathcal{U} \equiv (N, \mathbf{F})$$

$$\mathcal{F} \equiv (\mathbf{F}, c^2 \mathbb{P})$$

$$\begin{aligned} \frac{\partial N}{\partial t} + \nabla \cdot \mathbf{F} &= 0, \\ \frac{\partial \mathbf{F}}{\partial t} + c^2 \nabla \cdot \mathbb{P} &= 0 \end{aligned}$$



$$\frac{\partial \mathcal{U}}{\partial t} + \nabla \mathcal{F}(\mathcal{U}) = 0$$

**Explicit time discretisation:**

$$t^{n+1} = t^n + \Delta t$$

$$\mathcal{U}_l^{n+1} = \mathcal{U}_l^n + \frac{\Delta t}{\Delta x} (\mathcal{F}_{l-1/2}^n - \mathcal{F}_{l+1/2}^n)$$

(Implicit is also possible, has pros and cons)

# Intercell flux function

$$\mathcal{U}_l^{n+1} = \mathcal{U}_l^n + \frac{\Delta t}{\Delta x} (\mathcal{F}_{l-1/2}^n - \mathcal{F}_{l+1/2}^n)$$

$$\mathcal{U} \equiv (N, \mathbf{F})$$

$$\mathcal{F} \equiv (\mathbf{F}, c^2 \mathbb{P})$$

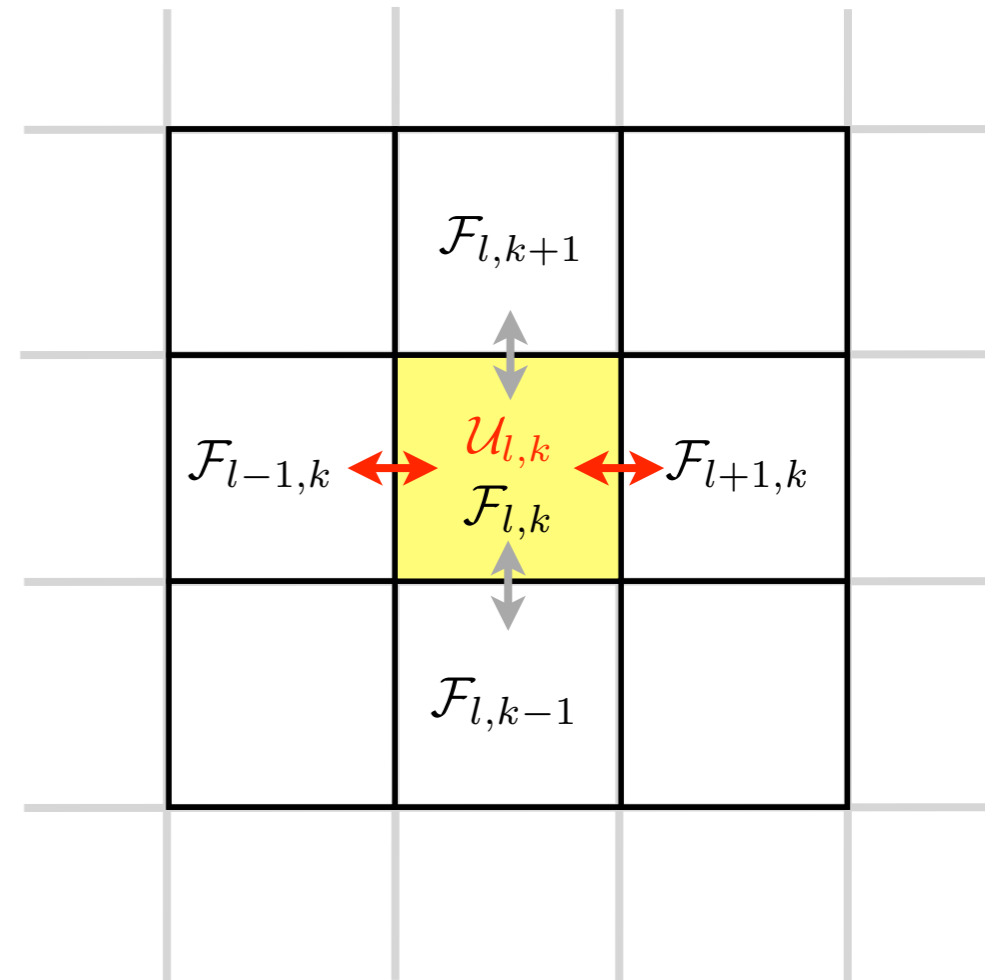
How do we pick  $\mathcal{F}_{l+1/2}$  ??

~~Simplest case is average between cells:~~

**Harten-Lax-van Leer - HLL:**

$$\mathcal{F}_{l+1/2} = \frac{\mathcal{F}_l + \mathcal{F}_{l+1}}{2}$$

$$\mathcal{F}_{l+1/2} = \frac{\lambda^+ \mathcal{F}_l - \lambda^- \mathcal{F}_{l+1} + \lambda^+ \lambda^- (\mathcal{U}_{l+1} - \mathcal{U}_l)}{\lambda^+ - \lambda^-}$$



$$\lambda^+ = \max(0, \lambda_l^{\max}, \lambda_{l+1}^{\max})$$

$$\lambda^- = \min(0, \lambda_l^{\min}, \lambda_{l+1}^{\min})$$

= angle-dependent 'speeds' = eigenvalues of  $\frac{\partial \mathcal{F}}{\partial \mathcal{U}}$

# a simpler flux function

**HLL:** 
$$\mathcal{F}_{l+1/2} = \frac{\lambda^+ \mathcal{F}_l - \lambda^- \mathcal{F}_{l+1} + \lambda^+ \lambda^- (\mathcal{U}_{l+1} - \mathcal{U}_l)}{\lambda^+ - \lambda^-}$$

A stable alternative is not to bother with eigenvalues at all. In the **Global Lax Friedrich function, or GLF**, we make the approximation that

$$\lambda^+ = -\lambda^- = c$$

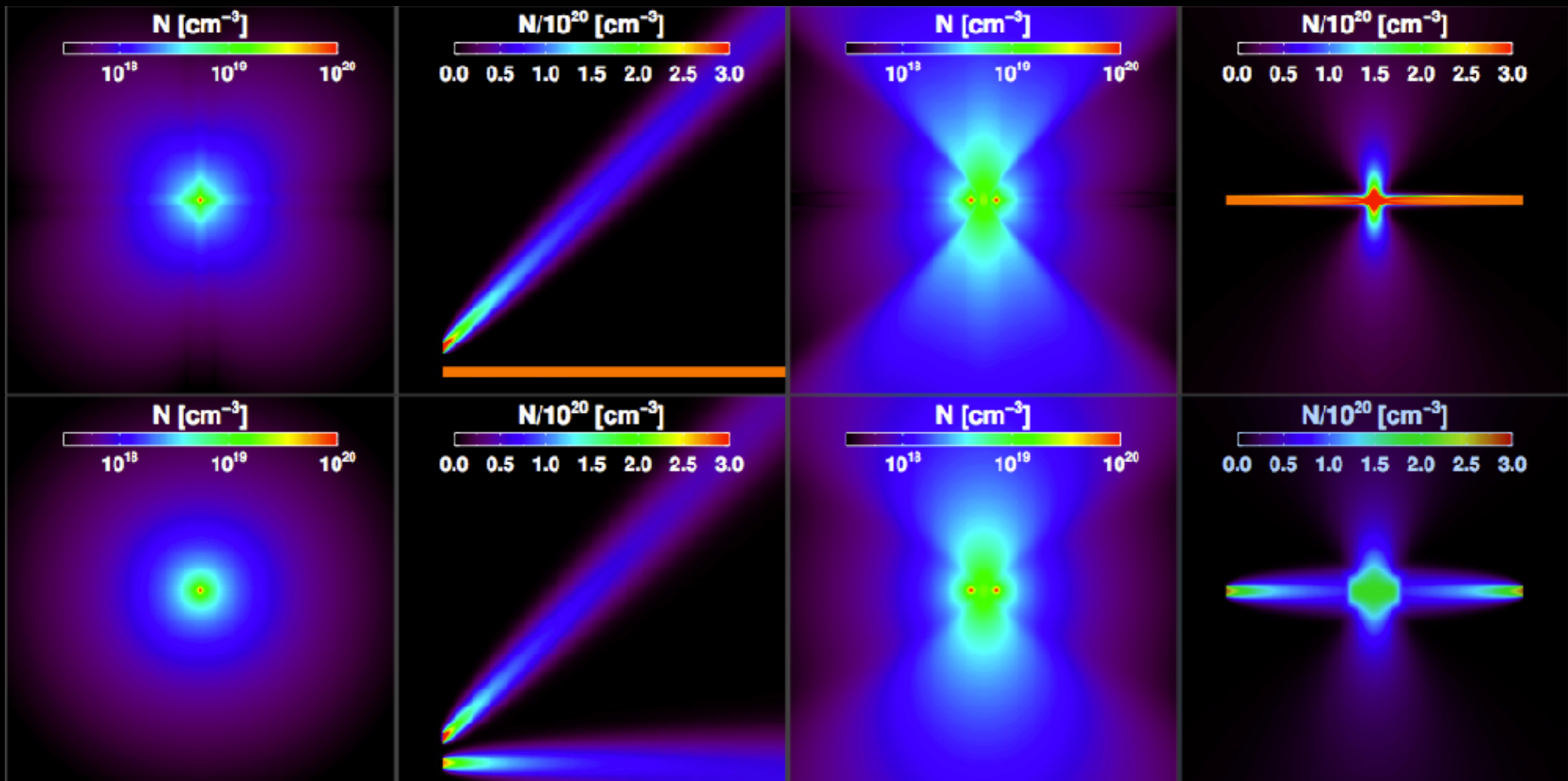
which gives

**GLF:** 
$$\mathcal{F}_{l+1/2} = \frac{\mathcal{F}_l + \mathcal{F}_{l+1}}{2} - \frac{c}{2} (\mathcal{U}_{l+1} - \mathcal{U}_l)$$

The resulting photon transport is **more diffusive than HLL**, which is both good and bad

# HLL vs GLF transport of photons

HLL



GLF

...I generally prefer GLF

# Thermochemistry

$$\frac{\partial N}{\partial t} + \nabla \cdot \mathbf{F} = - \sum_j^{\text{HI, HeI, HeII}} n_j \sigma_j c N + \dot{N}^* + \dot{N}^{rec}$$
$$\frac{\partial \mathbf{F}}{\partial t} + c^2 \nabla \cdot \mathbb{P} = - \sum_j^{\text{HI, HeI, HeII}} n_j \sigma_j c \mathbf{F}$$

---

$$\mathbb{P} = \mathbb{D}N$$
$$\frac{\partial \varepsilon}{\partial t} = \Lambda(\rho, \varepsilon, n_j, N_i)$$

- Hydro codes usually assume photoionisation equilibrium (PIE) where the gas ionisation fractions are a tabulated function of temperature and density
- Not ideal if we want to conserve photons
- Therefore we store and evolve ionisation fractions in each cell:  
 $x_{\text{HII}}, \quad x_{\text{HeII}}, \quad x_{\text{HeIII}}$
- Molecular hydrogen is coming soon

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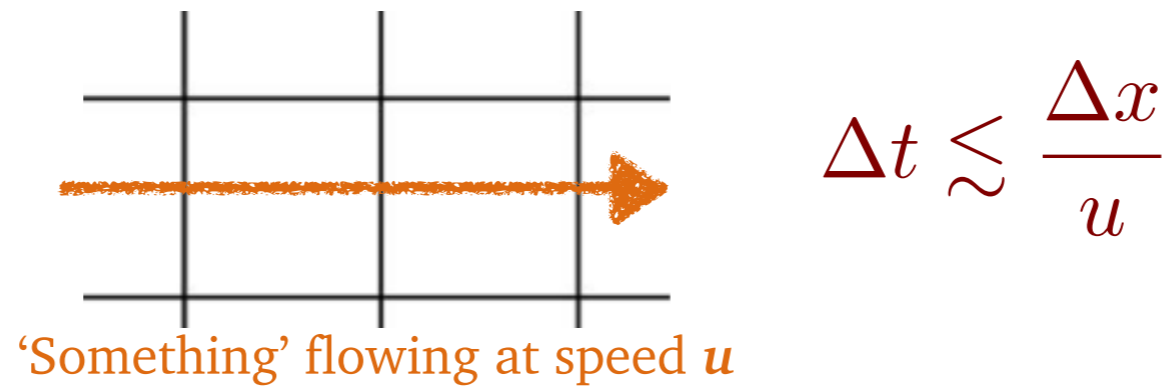
Tests

Science with RAMSES-RT



# The speed of light problem

The explicit advection of radiation across cells makes us slaves to the Courant condition



$$\Delta t_{\text{RT}} \sim \frac{\Delta x}{c} \sim \frac{\Delta t_{\text{HD}}}{1000}$$

# The reduced speed of light approximation (RSLA)

Gnedin & Abel 2001

To cheat death we use the reduced speed of light approximation (Gnedin & Abel 2001):

$$c_{\text{red}} = \frac{c}{1000} \quad \Rightarrow \quad \Delta t_{\text{RT}} \sim \frac{\Delta x}{c_{\text{red}}} \sim \Delta t_{\text{HD}}$$

⇒ Only ~2X runtime increase, compared to pure hydro

Not quite as bad as it sounds:

The dynamic speed in RHD simulations is that of *ionisation fronts*, not  $c$ .

We just want to get the front correct...

# Setting the reduced speed of light

Assuming a constant luminosity in a homogeneous medium...

Regime	$n_H$ (cm <sup>-3</sup> )	$\dot{N}$ (s <sup>-1</sup> )	$r_S$ (kpc)	$t_{\text{cross}}$ (Myr)	$t_{\text{rec}}$ (Myr)	$\tau_{\text{sim}}$ (Myr)	$f_{c, \text{min}}$
MW ISM	$10^{-1}$	$2 \times 10^{50}$	0.9	$3 \times 10^{-3}$	1.2	1	$3 \times 10^{-2}$
MW cloud	$10^2$	$2 \times 10^{48}$	$2 \times 10^{-3}$	$6 \times 10^{-6}$	$1 \times 10^{-3}$	0.1	$6 \times 10^{-4}$
Iliev tests 1, 2, 5	$10^{-3}$	$5 \times 10^{48}$	5.4	$2 \times 10^{-2}$	122.3	10	$2 \times 10^{-2}$
Iliev test 4	$10^{-4}$	$7 \times 10^{52}$	600	2	1200	0.05	1

Reionisation

$$f_c = \min(1; \sim 10 \times t_{\text{cross}} / \tau_{\text{sim}})$$

These are suggestive values...

Light speed convergence tests are the best final verdict.

Great for ISM and CGM simulations, but death for reionisation (of cosmological voids) 😞

# The variable speed of light approximation (VSLA)

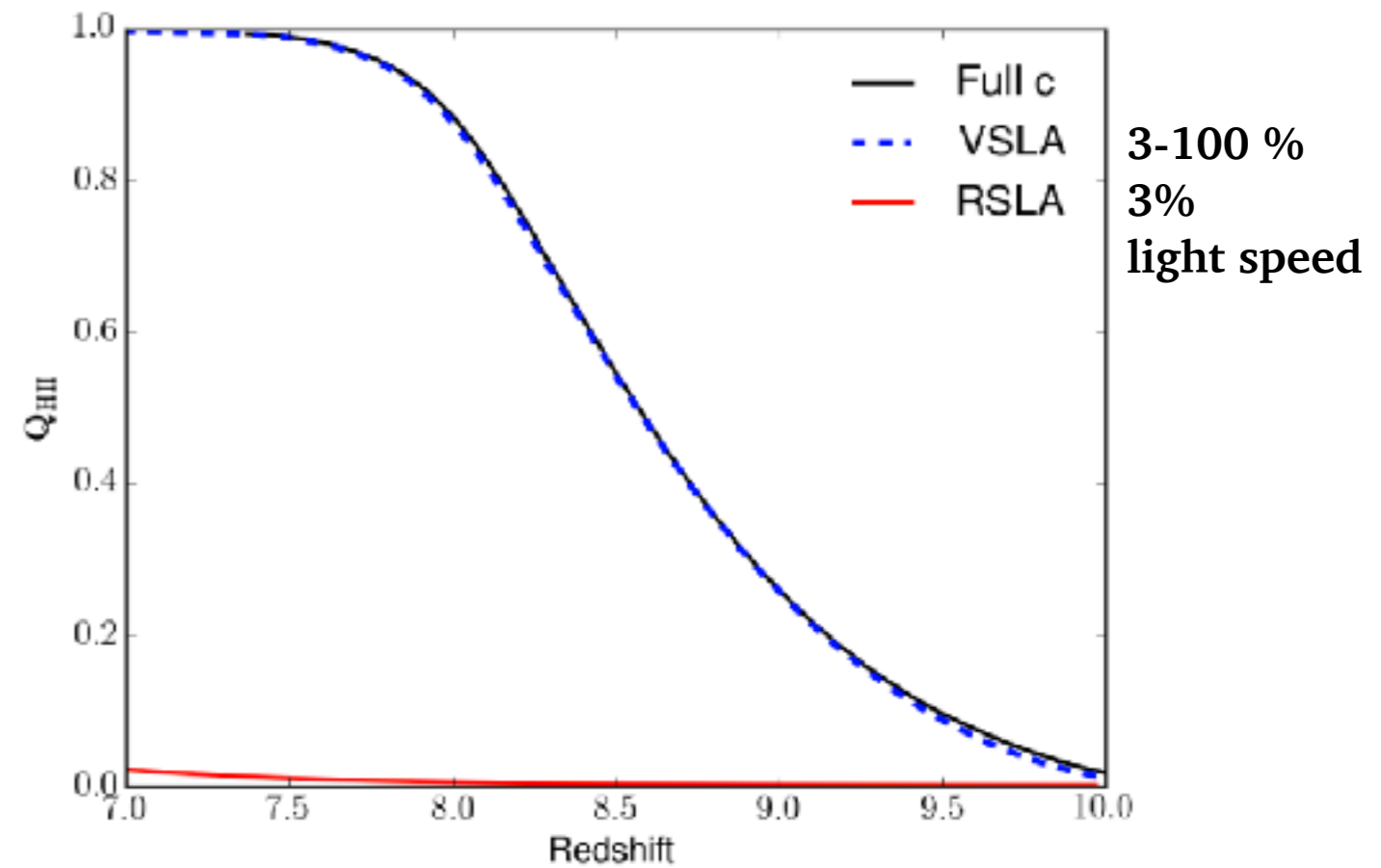
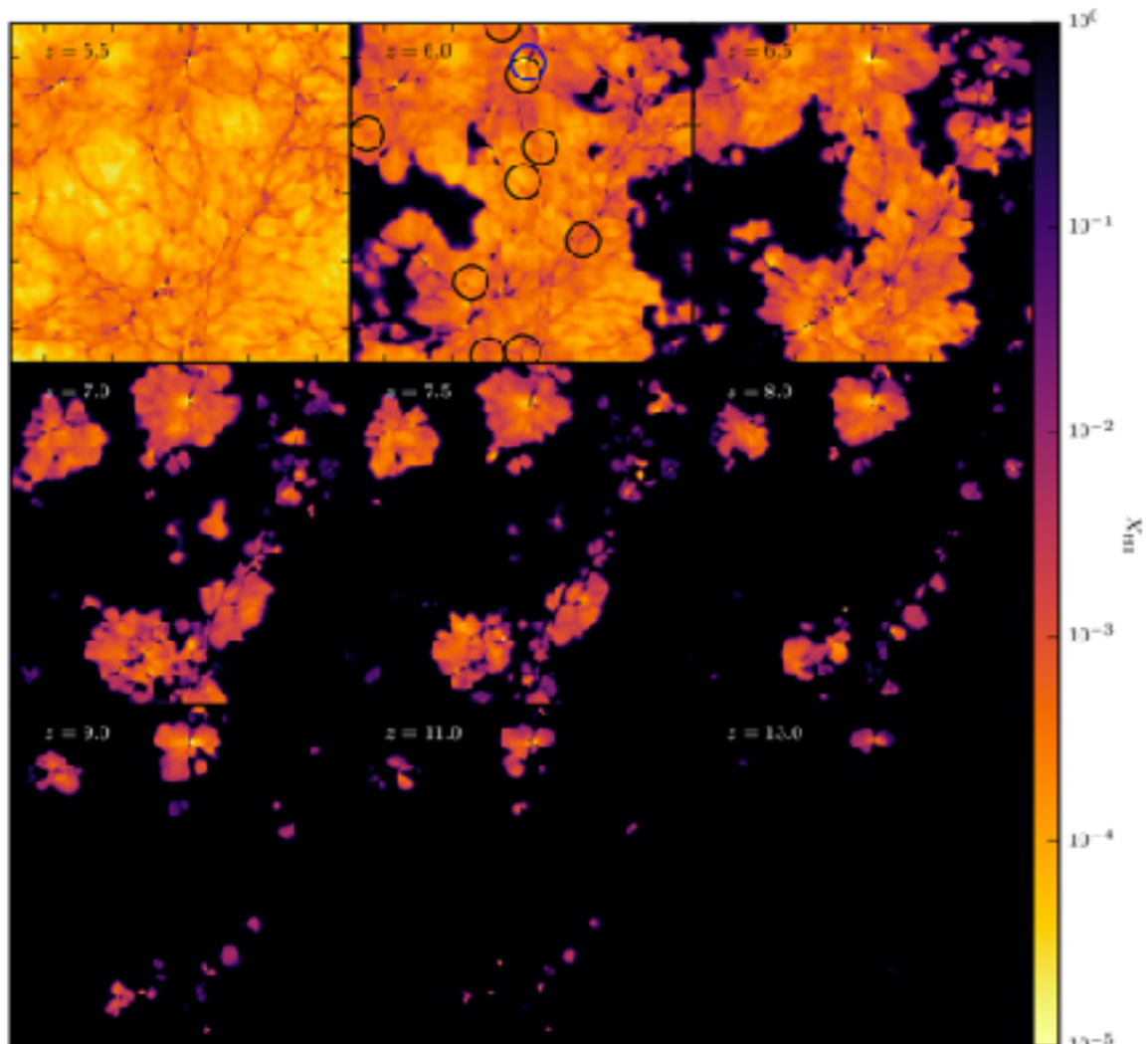
Katz et al. 2017

- A  $\sim$ full light speed matters most in diffuse ‘voids’ where I-fronts are *fast*.
- Taking advantage of this, Harley Katz implemented in Ramses-RT a **variable light-speed**, making reionisation simulations feasible with RAMSES-RT
- Here, we use a slow light speed at the finest AMR level and increase with each coarser level, towards a full light speed in the coarsest ‘voids’ (where there are ideally few cells).

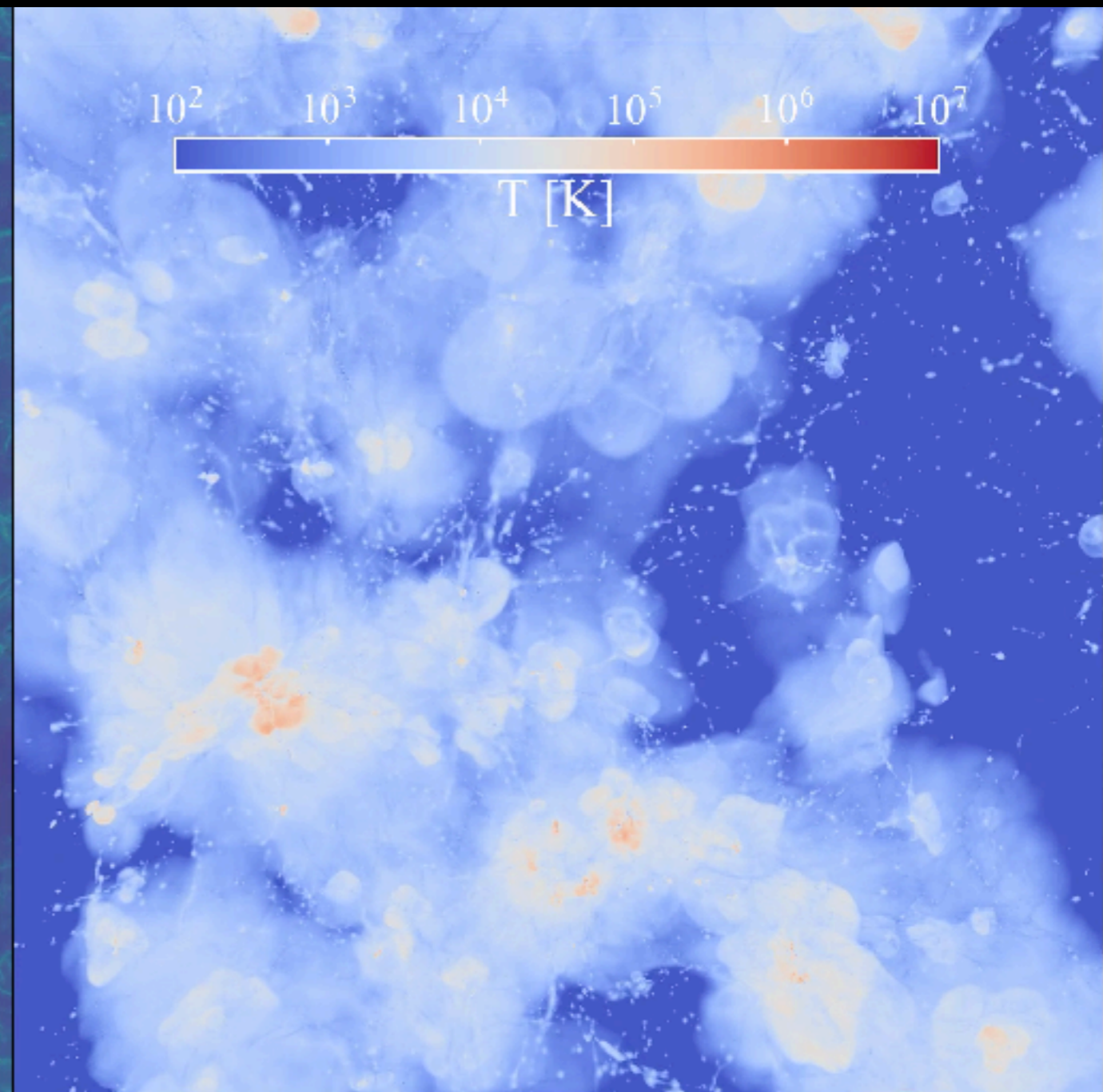
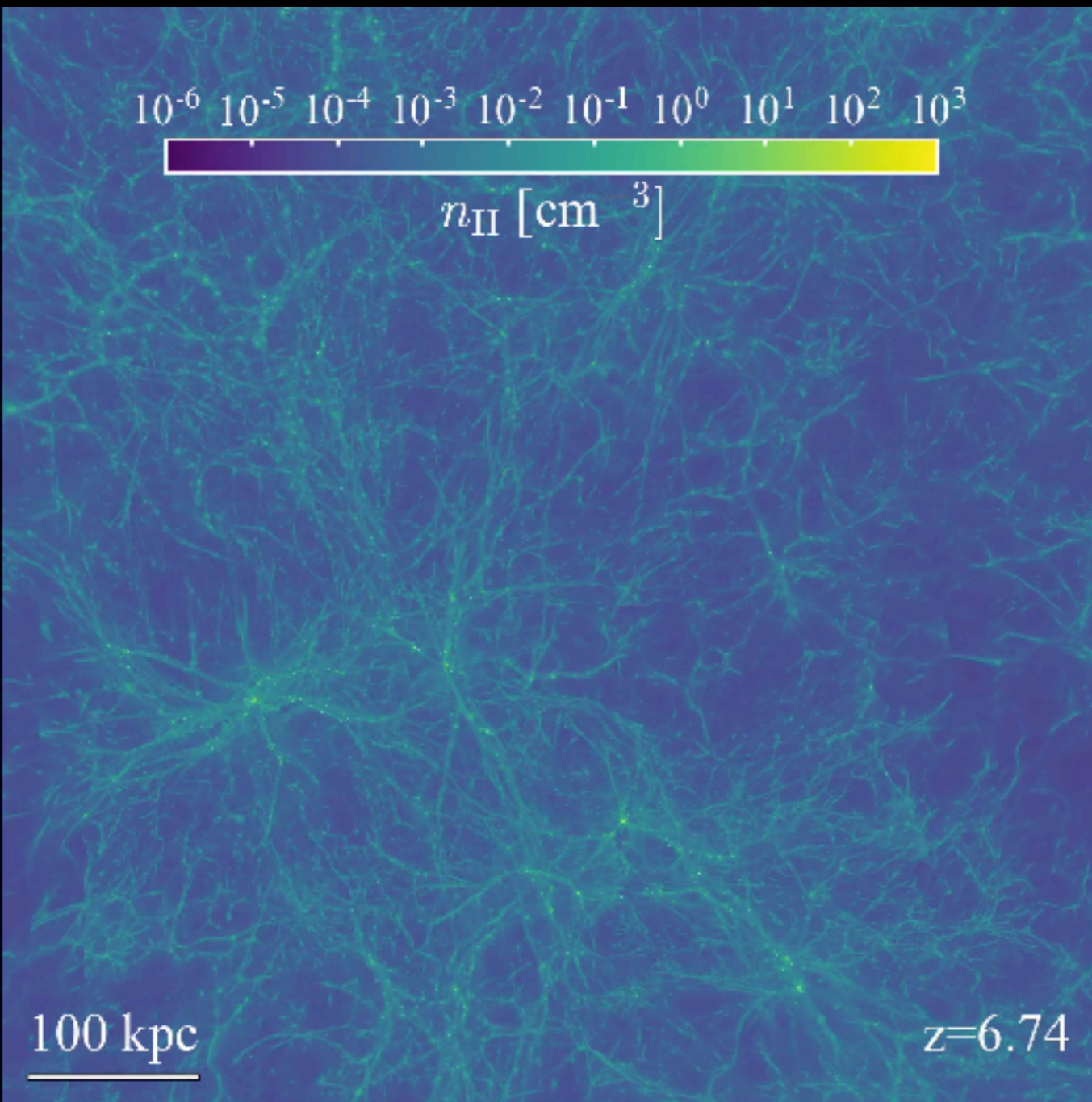


# The variable speed of light approximation (VSLA)

From Katz et al. 2017



# Using the VSLA for high resolution reionisation simulations



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# RHD: Coupling hydrodynamics and RT

HD equations solved in Ramses:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

$$\frac{\partial}{\partial t} (\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) + \nabla p = \rho \nabla \phi$$

$$\frac{\partial \mathcal{E}}{\partial t} + \nabla \cdot (\mathcal{E} + p) \mathbf{u} = -\rho \mathbf{u} \cdot \nabla \phi + \Lambda(\rho, \varepsilon)$$

$$p = (\gamma - 1)\varepsilon$$

$$\mathcal{E} = \frac{1}{2}\rho u^2 + \varepsilon$$

RT equations:

$$\frac{\partial N}{\partial t} + \nabla \cdot \mathbf{F} = - \sum_j^{\text{HI, HeI, HeII}} n_j \sigma_j c N + \dot{N}^* + \dot{N}^{rec}$$

$$\frac{\partial \mathbf{F}}{\partial t} + c^2 \nabla \cdot \mathbb{P} = - \sum_j^{\text{HI, HeI, HeII}} n_j \sigma_j c \mathbf{F}$$

$$\mathbb{P} = \mathbb{D}N$$

$$\frac{\partial \varepsilon}{\partial t} = \Lambda(\rho, \varepsilon, n_j, N_i)$$

HD and RT couple where photons interact with gas

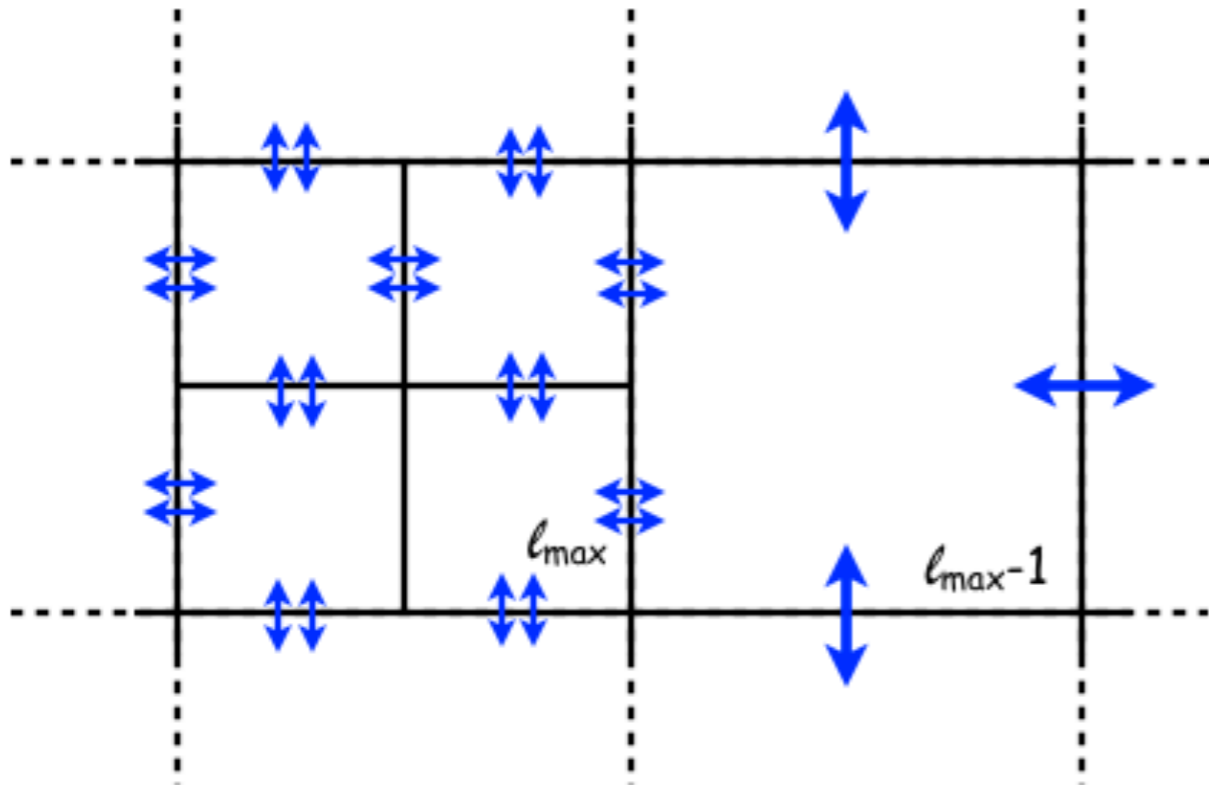
Considerations:

- We should explicitly keep track of ionization fractions,
  - ➔ Non-equilibrium thermochemistry of hydrogen and helium, with local photoheating
- ...which should be done on the shorter RT timescale
  - ➔ HD is left virtually unchanged, and RT is subcycled after HD timestep = **Operator splitting**
- ...but AMR subcycling makes this tricky



# The AMR timestep in Ramses

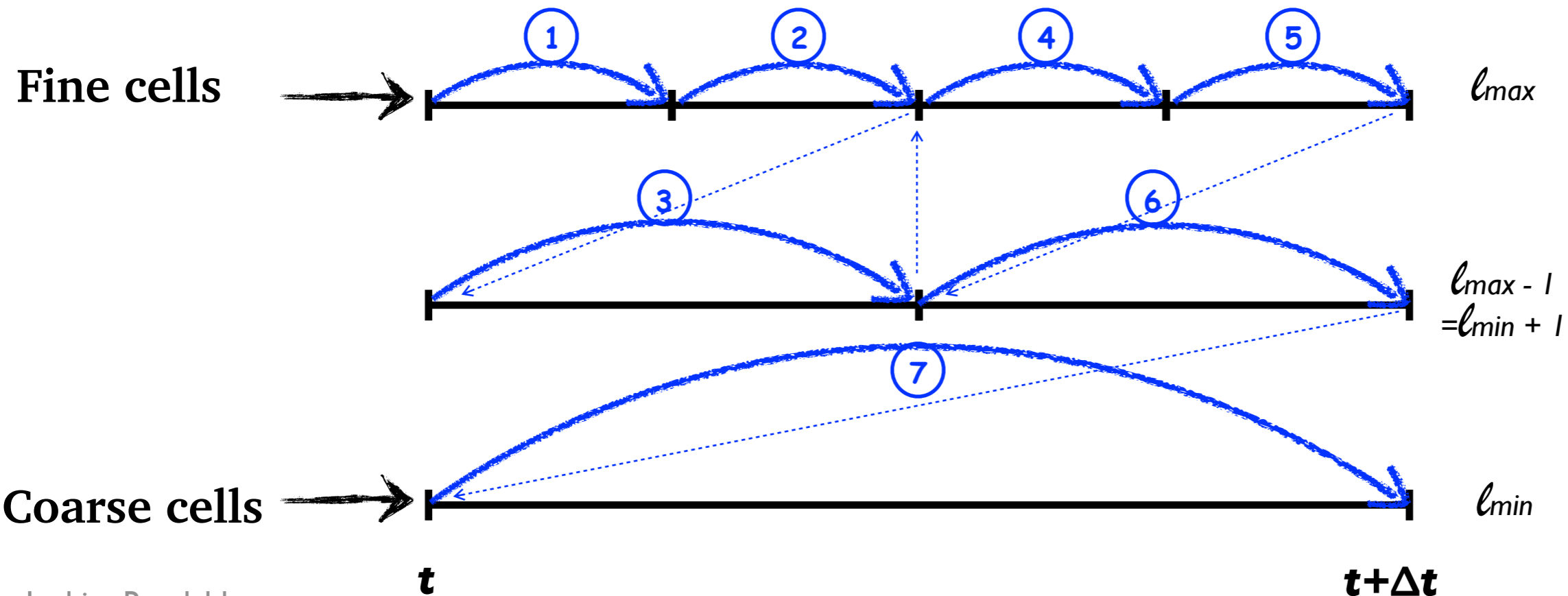
## Adaptive resolution both in space and time



The timestep length is a cell crossing time (Courant condition):

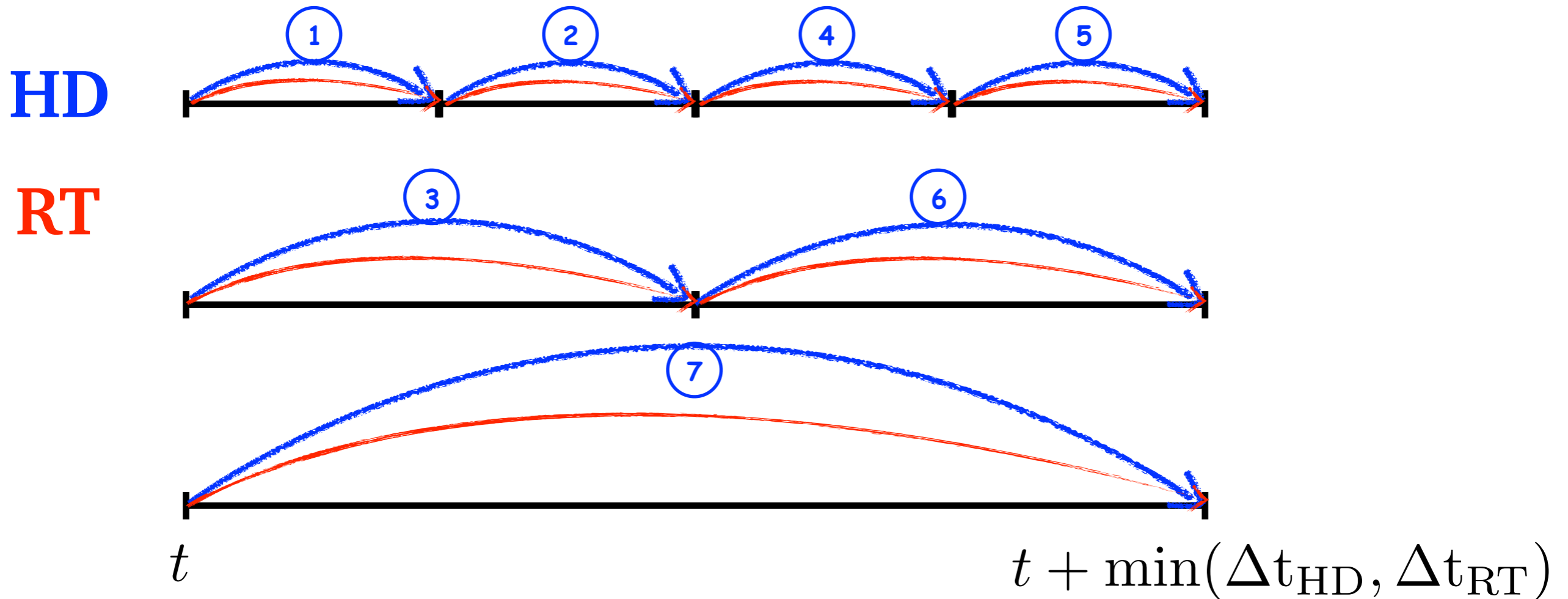
$$\Delta t_l \approx \frac{\Delta x_l}{u}$$

→ The levels are multisteped in time when solving HD -- one coarse step per two fine steps



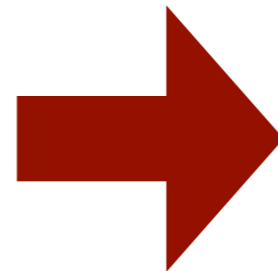
# Where does the RT fit in?

Approach a): Do a single RT step after HD on each level



$$\Delta t_{\text{HD}} \sim \frac{\Delta x}{u}$$

$$\Delta t_{\text{RT}} \sim \frac{\Delta x}{c} \sim \frac{\Delta t_{\text{HD}}}{1000}$$



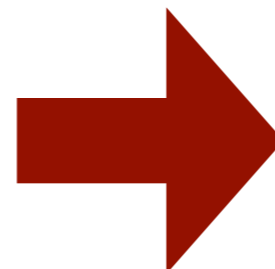
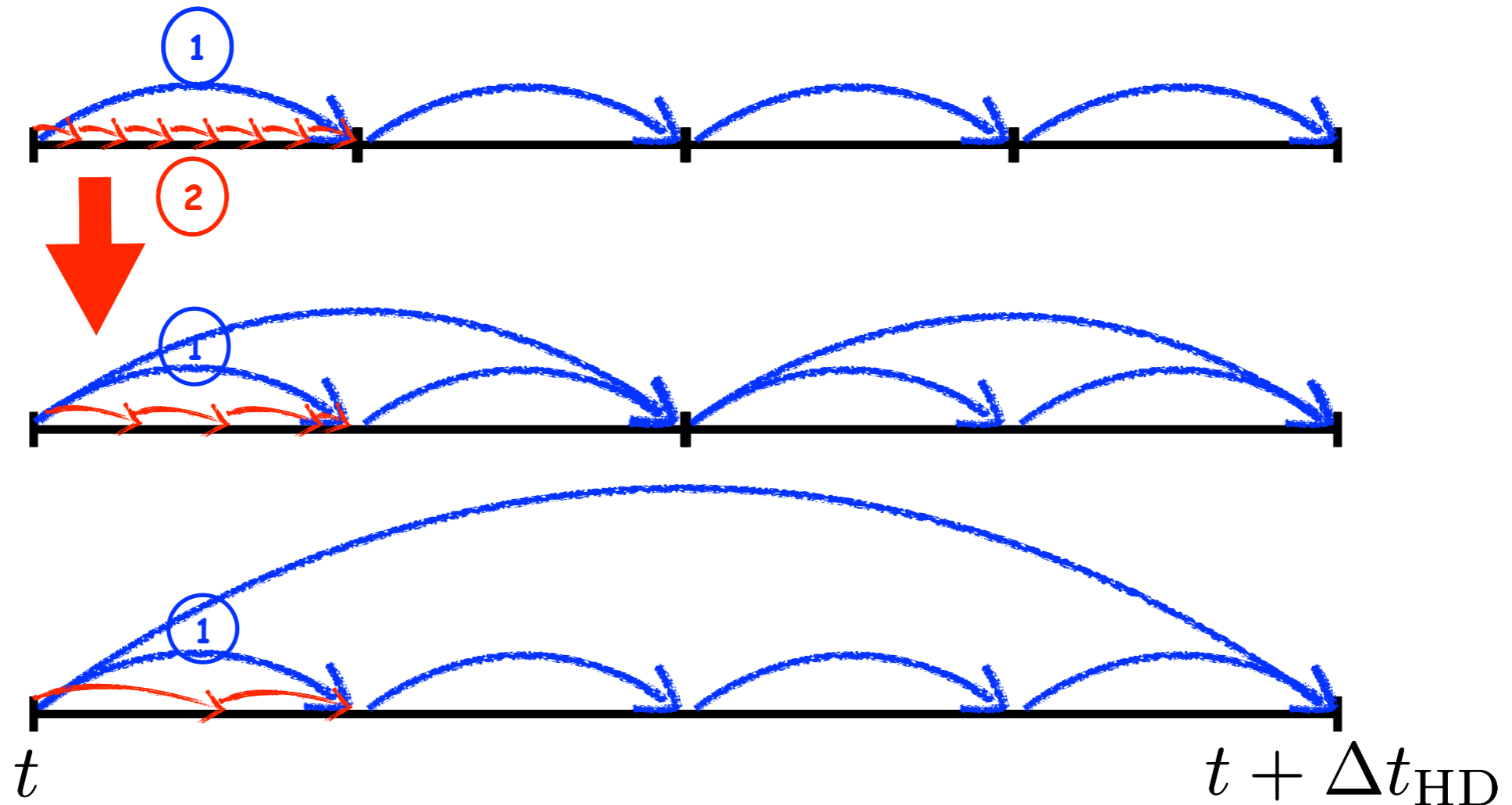
Orders of magnitude shortening of the HD timestep

# Where does the RT fit in?

Approach b): Many RT steps after HD on each level

HD

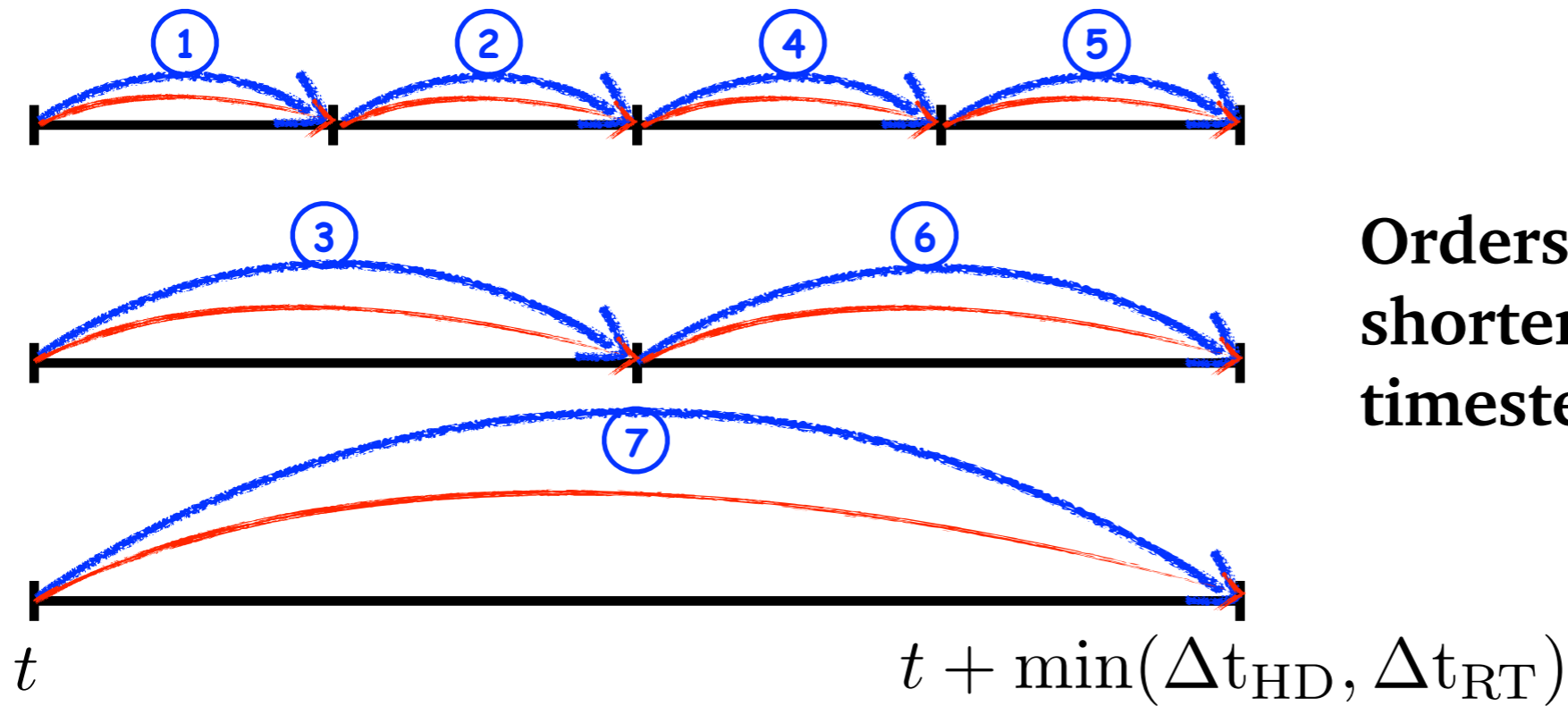
RT



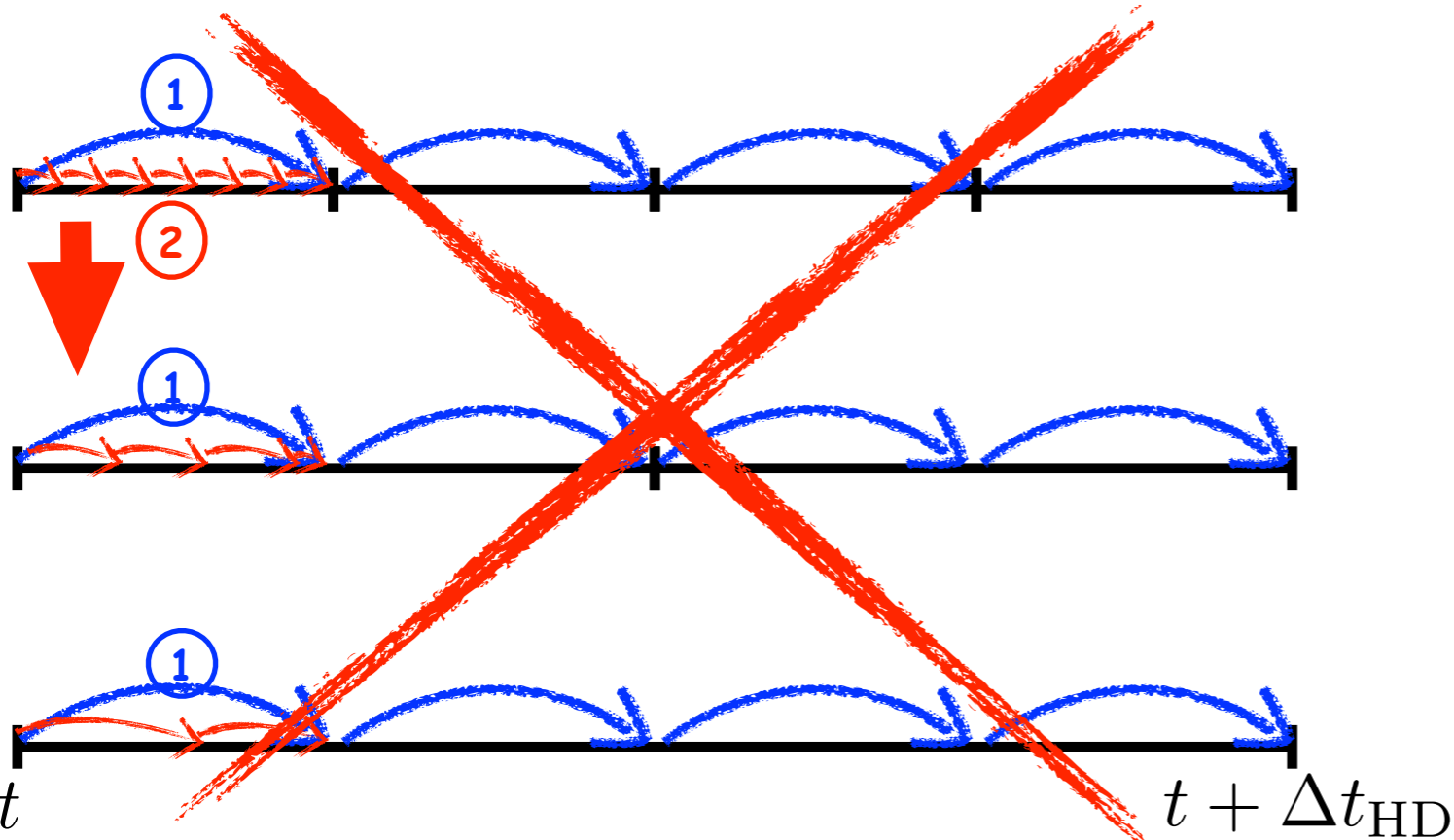
No HD multisteping  
(no time refinement)

# Where does the RT fit in?

a) vs b)



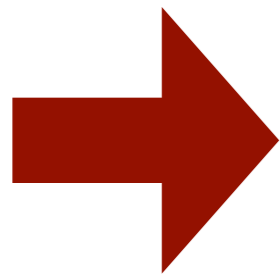
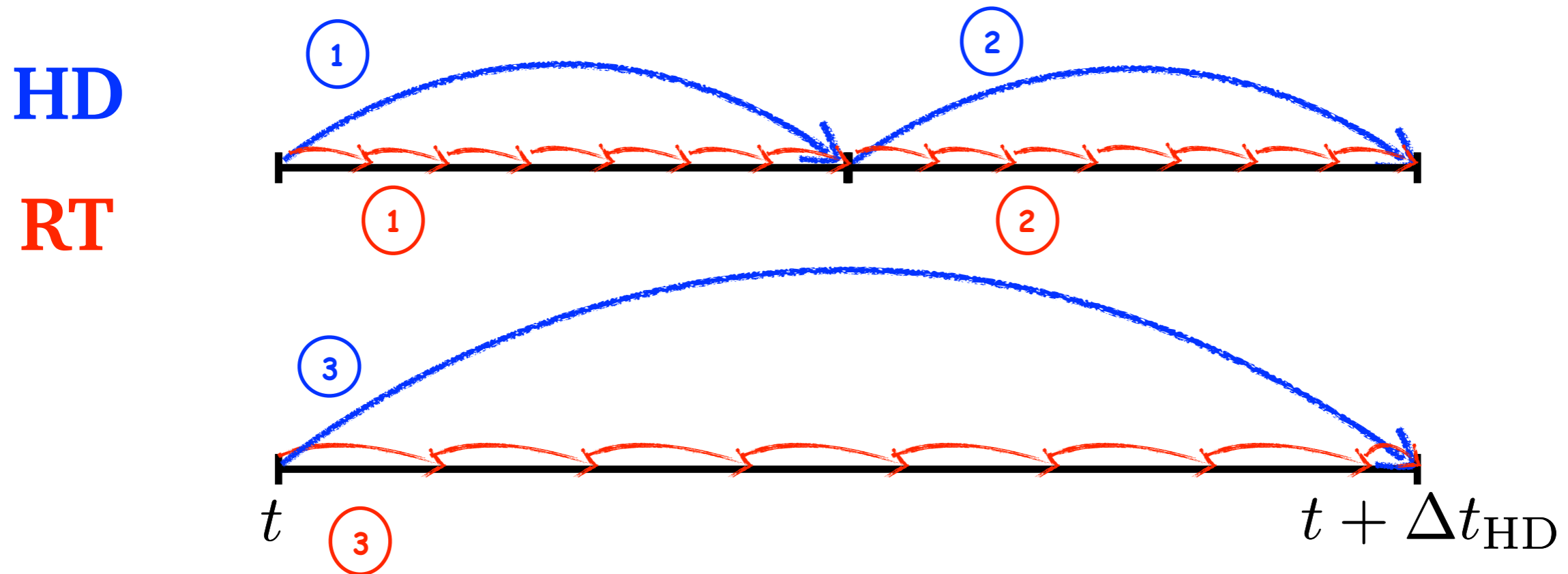
Orders of magnitude shortening of the HD timestep



No HD multisteping  
(no time refinement)

# Plan c: rt\_subcycle

Approach c): Many RT steps after HD on each level and lose photons across boundaries

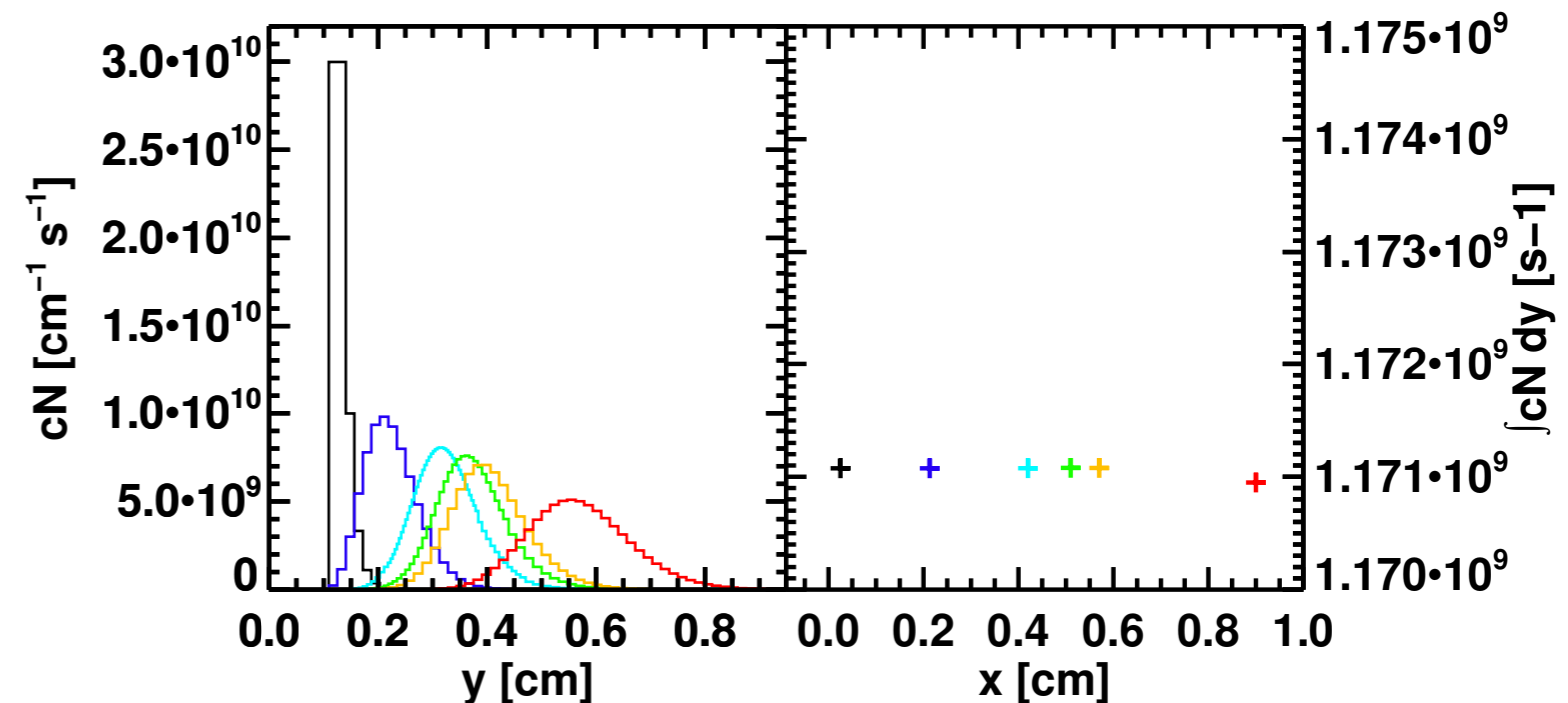
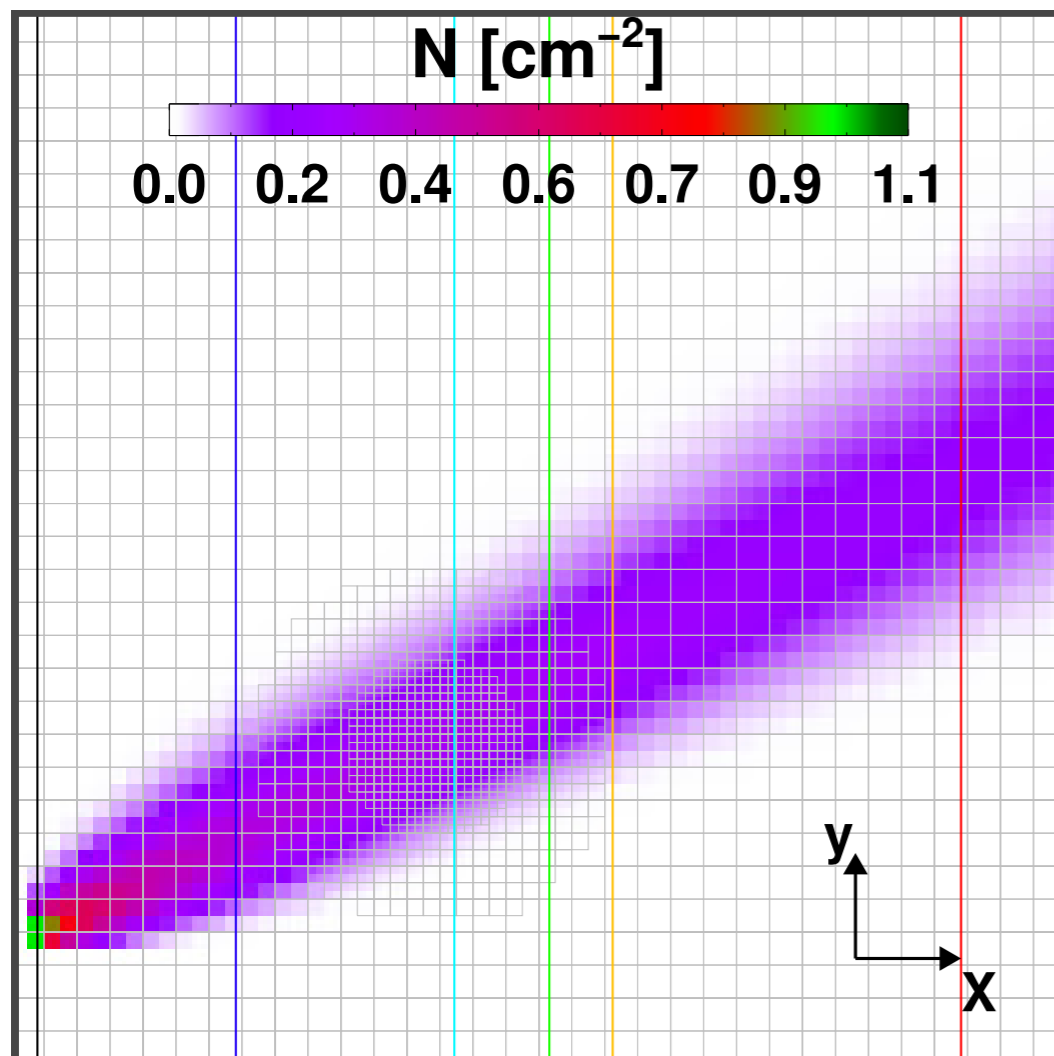


Keep HD multistepping and time refinement, while subcycling RT

Sacrifice perfect photon conservation across level boundaries

# Radiation transport across an AMR grid

- RT variables are stored in cells, in just the same way as hydro variables
- So there's not much to do here but just take advantage of native AMR strategy and interpolation schemes already in place in Ramses
- In the same way, the RHD is fully MPI parallel, as is the rest of RAMSES



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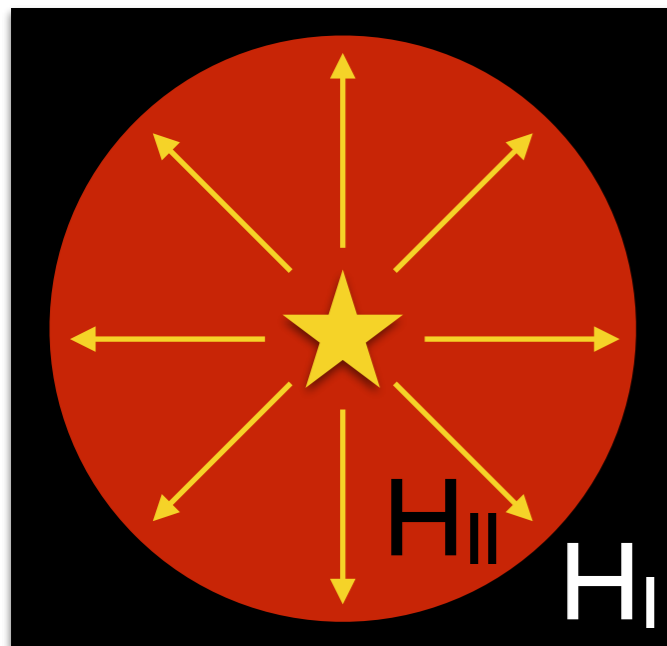
Radiation pressure and multi-scattering

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Science with RAMSES-RT

# Radiation pressure

- May play a role in suppressing star formation and even generating outflows
- Cosmo simulations often include radiation pressure as a sub-grid recipe:



Stellar (UV) luminosity

UV optical depth

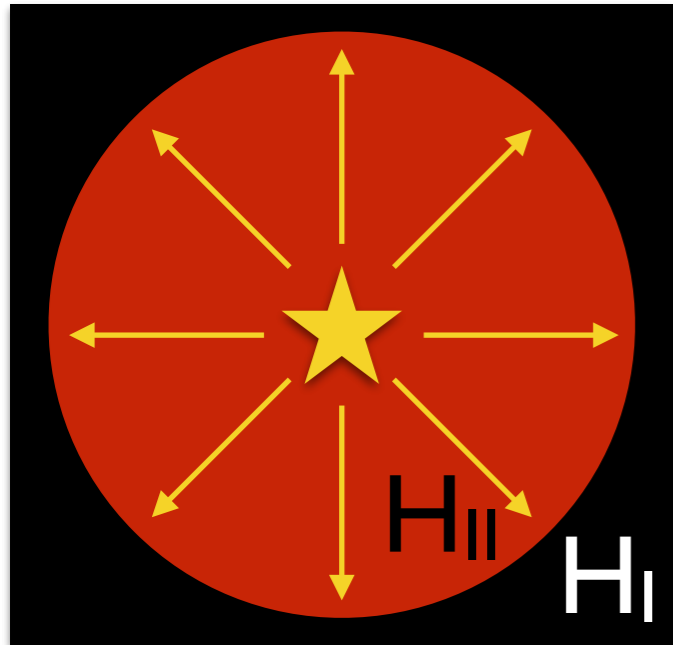
$$\dot{p}_{\text{rad}} = \frac{L_{\text{UV}}}{c} (1 - e^{-\tau_{\text{UV}}}) \approx \frac{L_{\text{UV}}}{c}$$

Momentum absorption



# Radiation pressure

- We account for the absorbed momentum in every cell in each step:



~~$$\dot{p}_{\text{rad}} = \frac{L_{\text{UV}}}{c} (1 - \langle \cos \theta \rangle) \approx \frac{L_{\text{UV}}}{c}$$~~

$$\frac{\partial \rho v}{\partial t} = \sum_i^{\text{groups}} \frac{F_i}{c} \left( \sum_j^{\text{H I, He I, He II}} \sigma_{ij} n_j \right)$$

# Multi-scattering

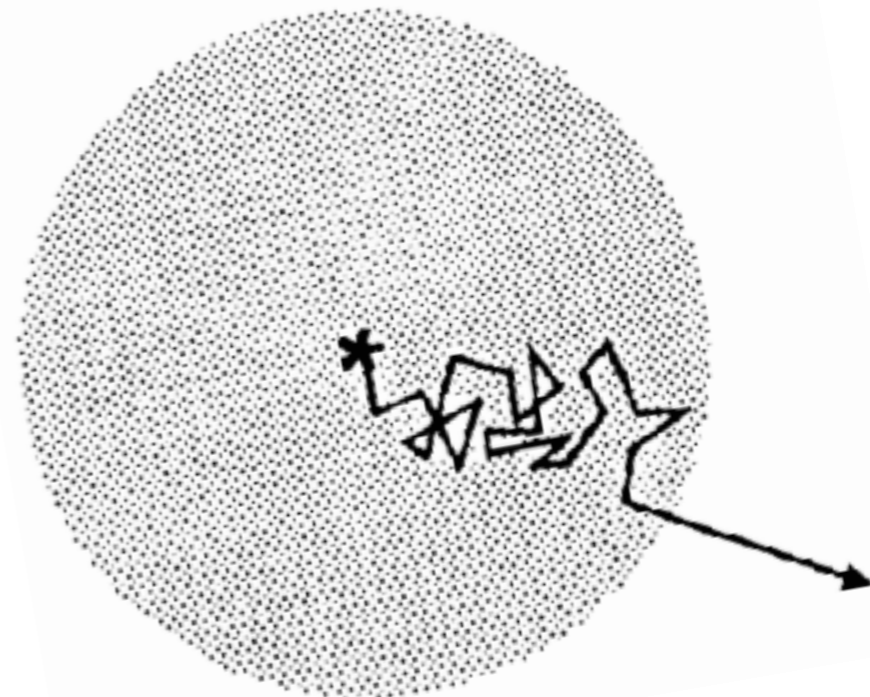
- Recently, we added dust absorption and scattering

$$\frac{\partial E_{\text{IR}}}{\partial t} + \nabla \cdot \mathbf{F}_{\text{IR}} = \kappa_{\text{P}} \rho (caT^4 - \tilde{c}E_{\text{IR}}) + \dot{E}_{\text{IR}}$$
$$\frac{\partial \mathbf{F}_{\text{IR}}}{\partial t} + \tilde{c}^2 \nabla \cdot \mathbb{P}_{\text{IR}} = -\kappa_{\text{R}} \rho \tilde{c} \mathbf{F}_{\text{IR}}$$

- IR radiation pressure on dust may be important in galaxy evolution because of **multi-scattering** pressure boost

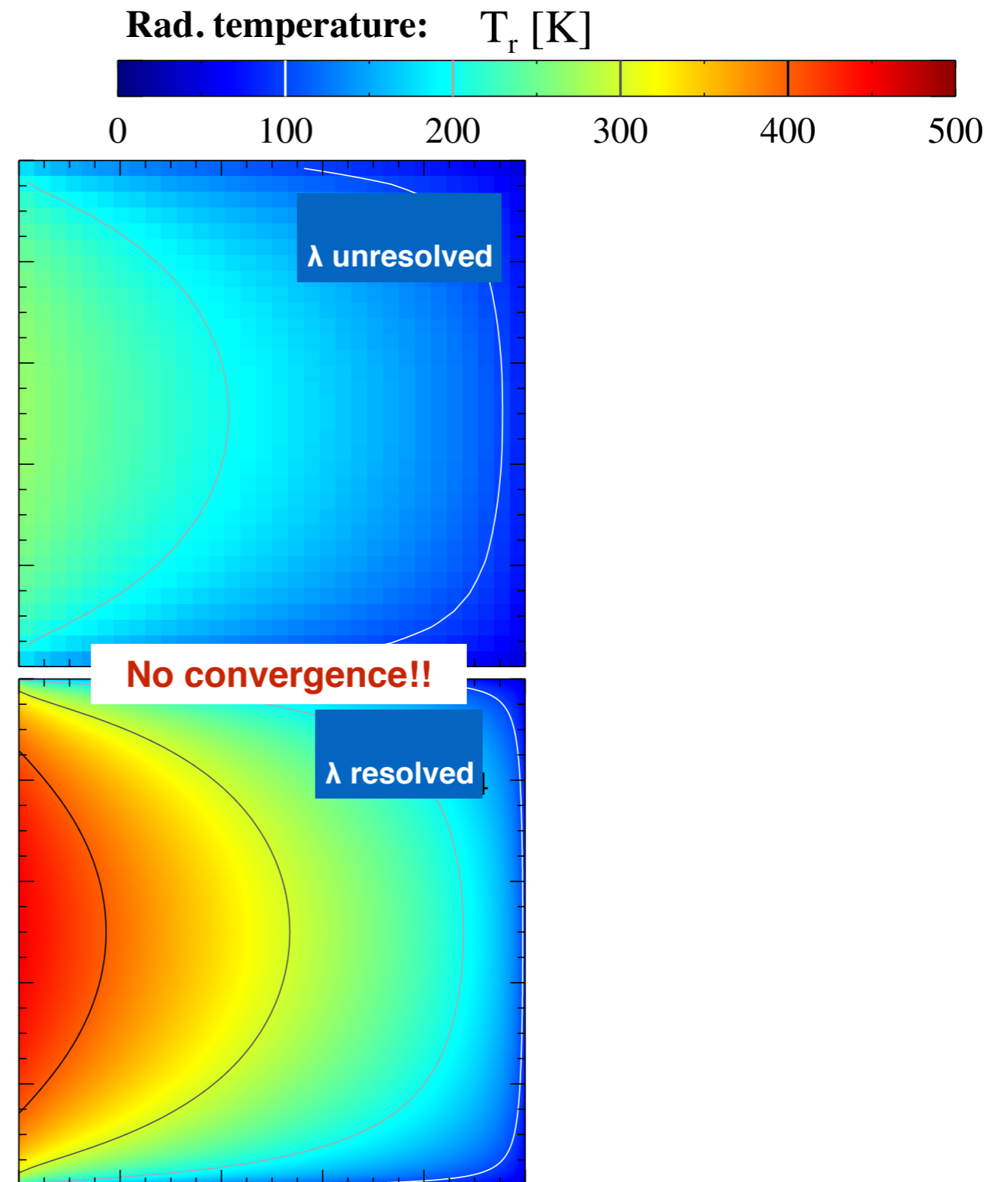
$$\dot{p}_{\text{rad}} = \frac{L_{\text{Opt}}}{c} \tau_{\text{IR}}$$

- For implementation details, see Rosdahl & Teyssier 2015



# Diffusion of radiation in optically thick gas

- The challenge is to model IR radiation in both the optically thin and thick limits
- The de-facto mean free path is
$$\lambda_{\text{eff}} = \max(\lambda, \Delta x)$$



From Rosdahl & Teyssier, 2015

# Diffusion of radiation in optically thick gas

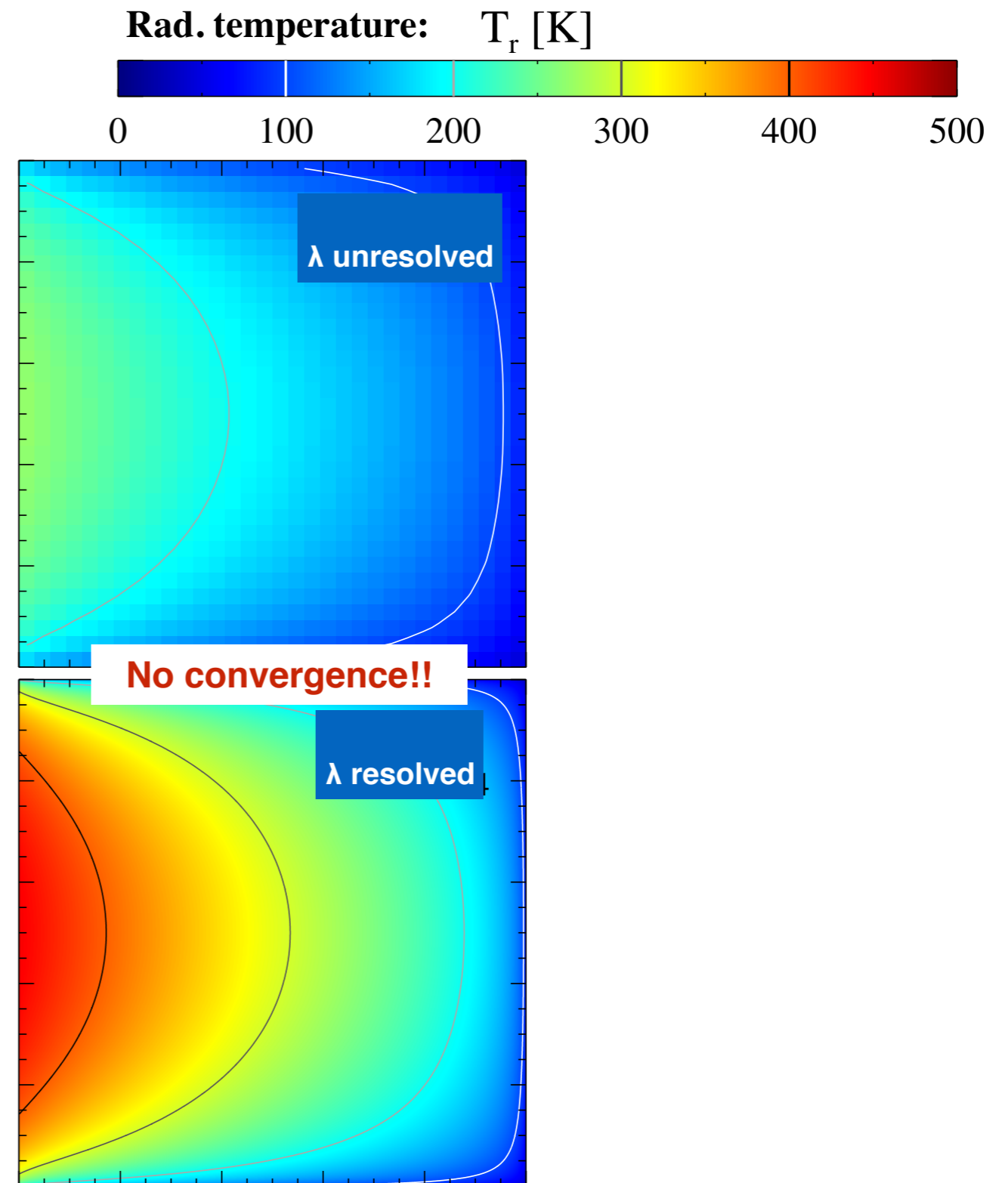
We divide the IR photons into **trapped** and **streaming** components in each cell, with

$$f_{\text{trapped}} = \exp\left(\frac{2}{3\tau_{\text{cell}}}\right)$$

We then recover the correct diffusion limit when the mean free path is unresolved, i.e.

$$\mathbf{F}_{\text{rad}} = -\frac{c\lambda}{3}\nabla E_{\text{rad}}$$

- ➔ Radiative transfer is accurate both in the diffusion and free-streaming limits (but at a reduced light speed)



From Rosdahl & Teyssier, 2015

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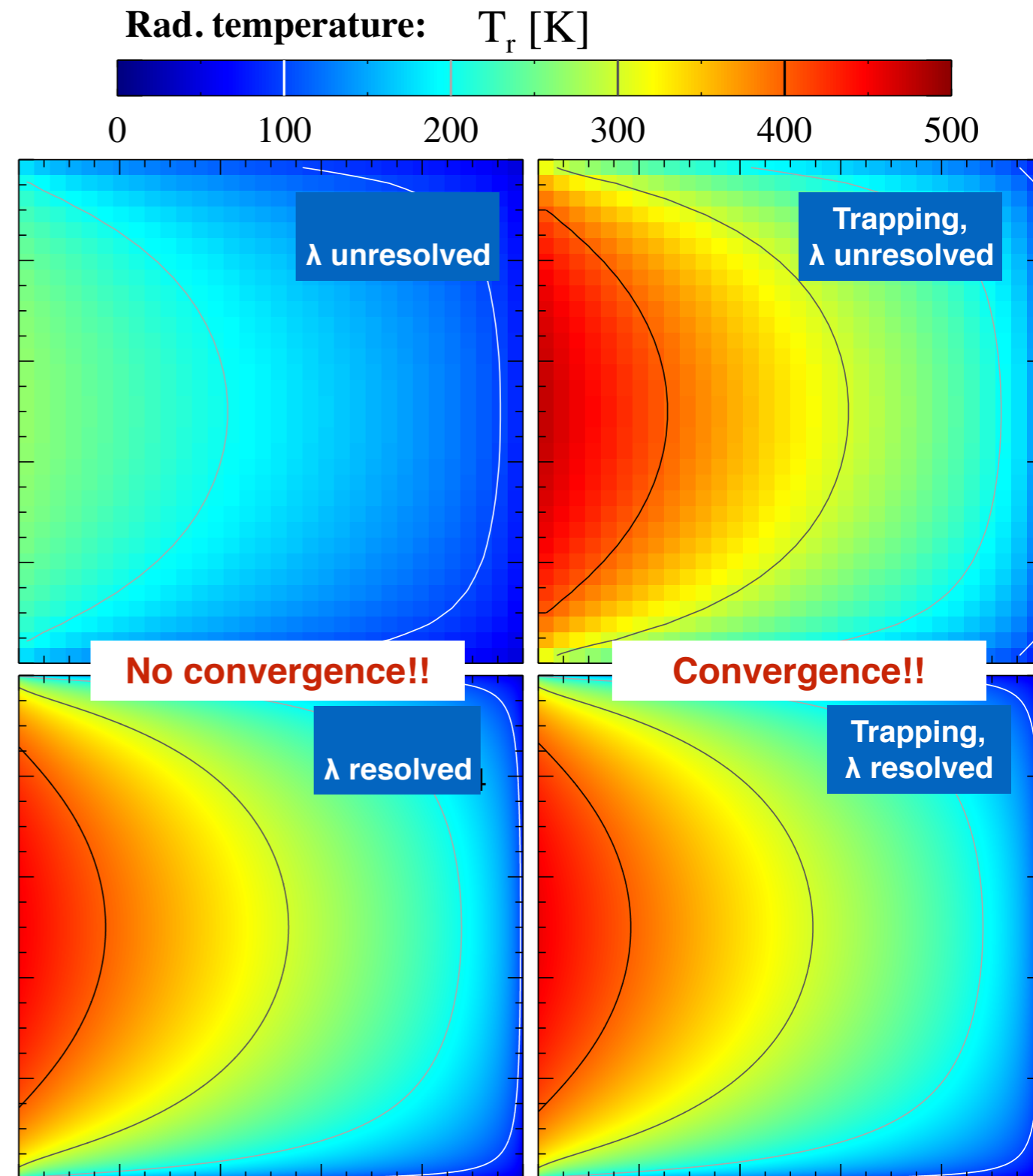
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# Validation tests for RamsesRT

## Thermochemistry tests

- Convergence of temperature and ionization states
- Stability
- Tests turn out ok

## Iliev et al's 'RT codes comparison project'

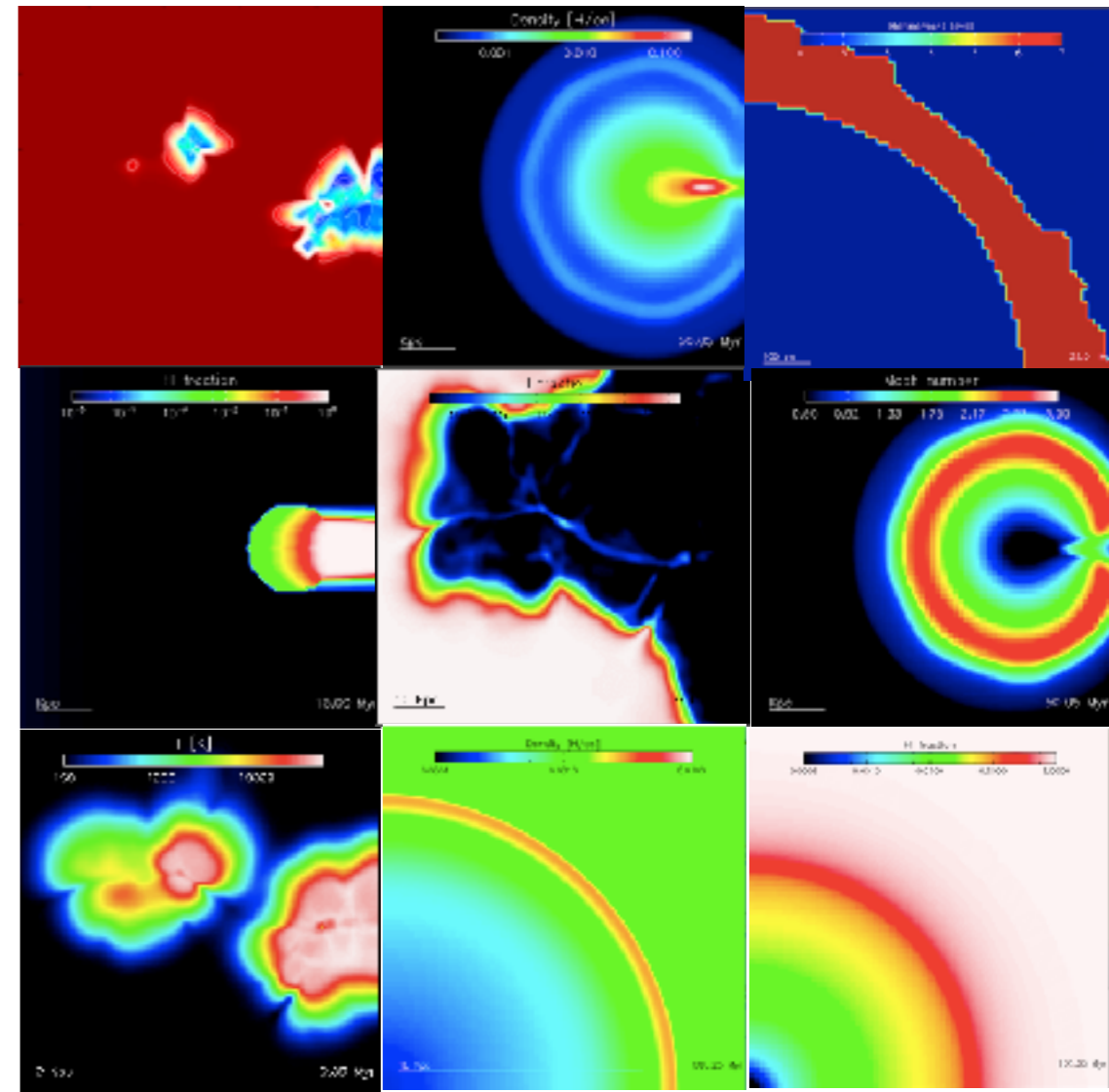
- Compare against other codes results

### Pure RT:

- 1) Isothermal HII region expansion
- 2) HII region expansion with cooling
- 3) Shadow test
- 4) Ionizing a cosmological volume

### RHD:

- 5) HII D-type expansion
  - 6) HII expansion in a  $r^{-2}$  density profile
  - 7) Photo-evaporation of a dense clump
- Comparisons good, except for 4)

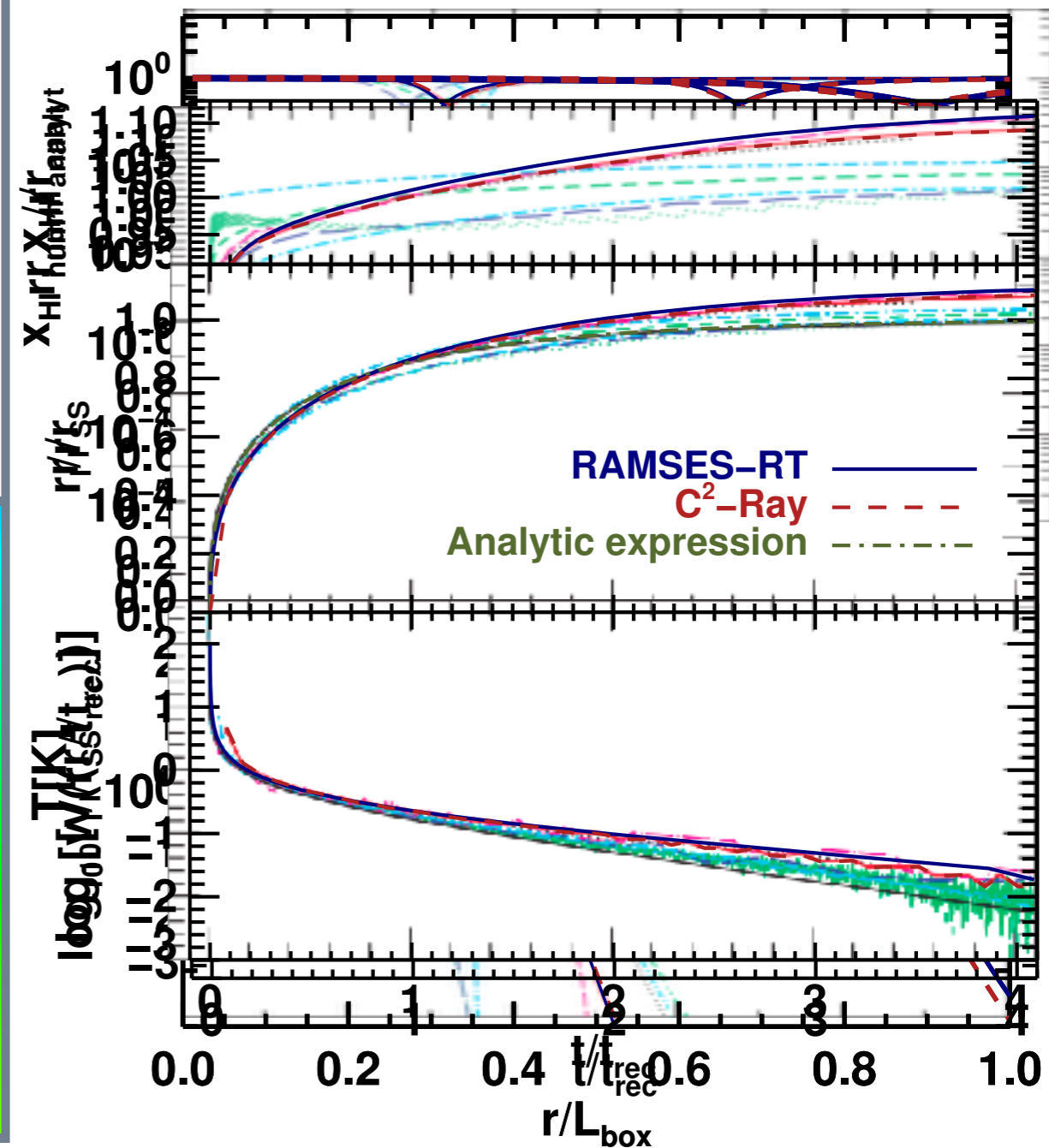
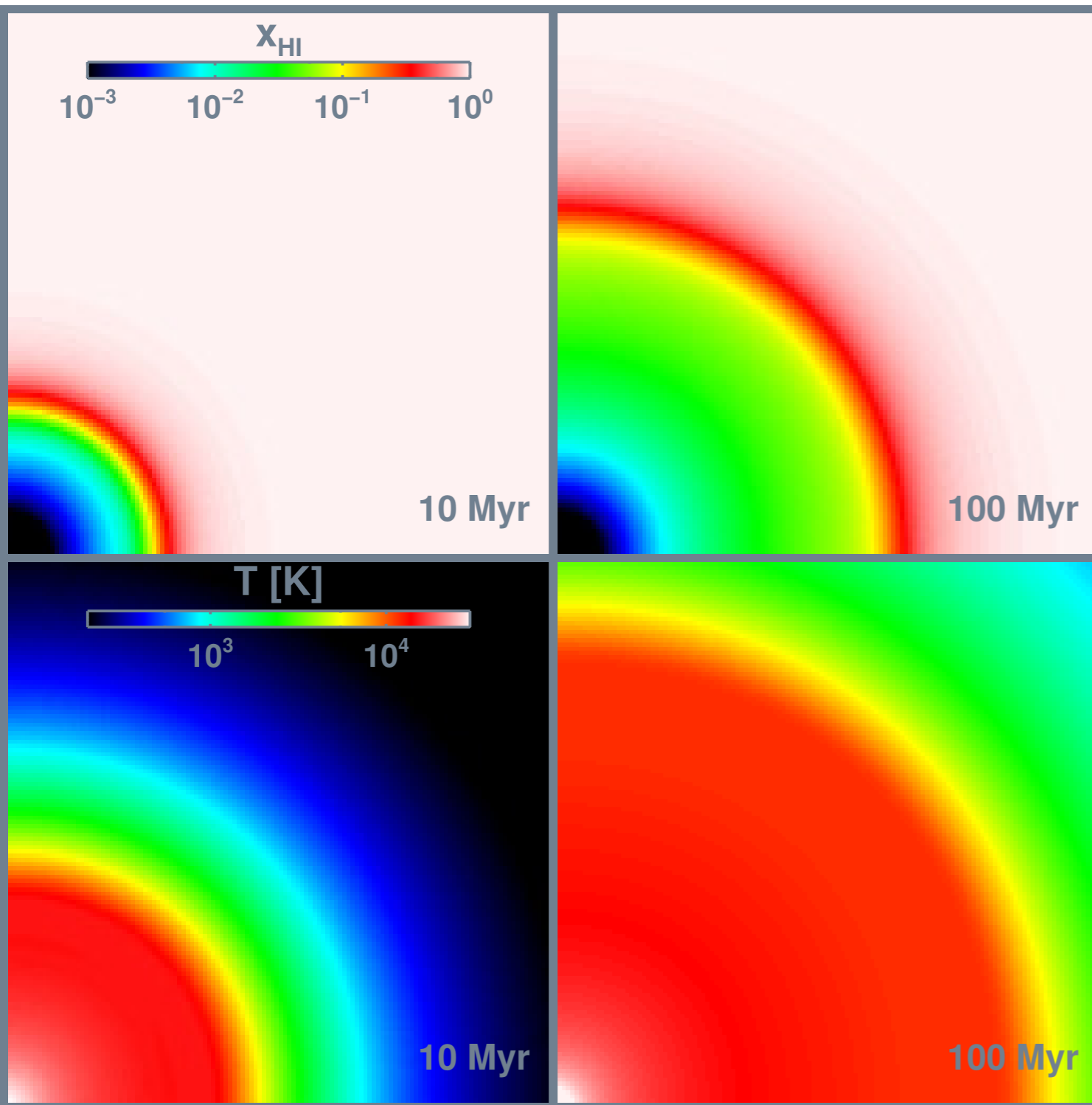


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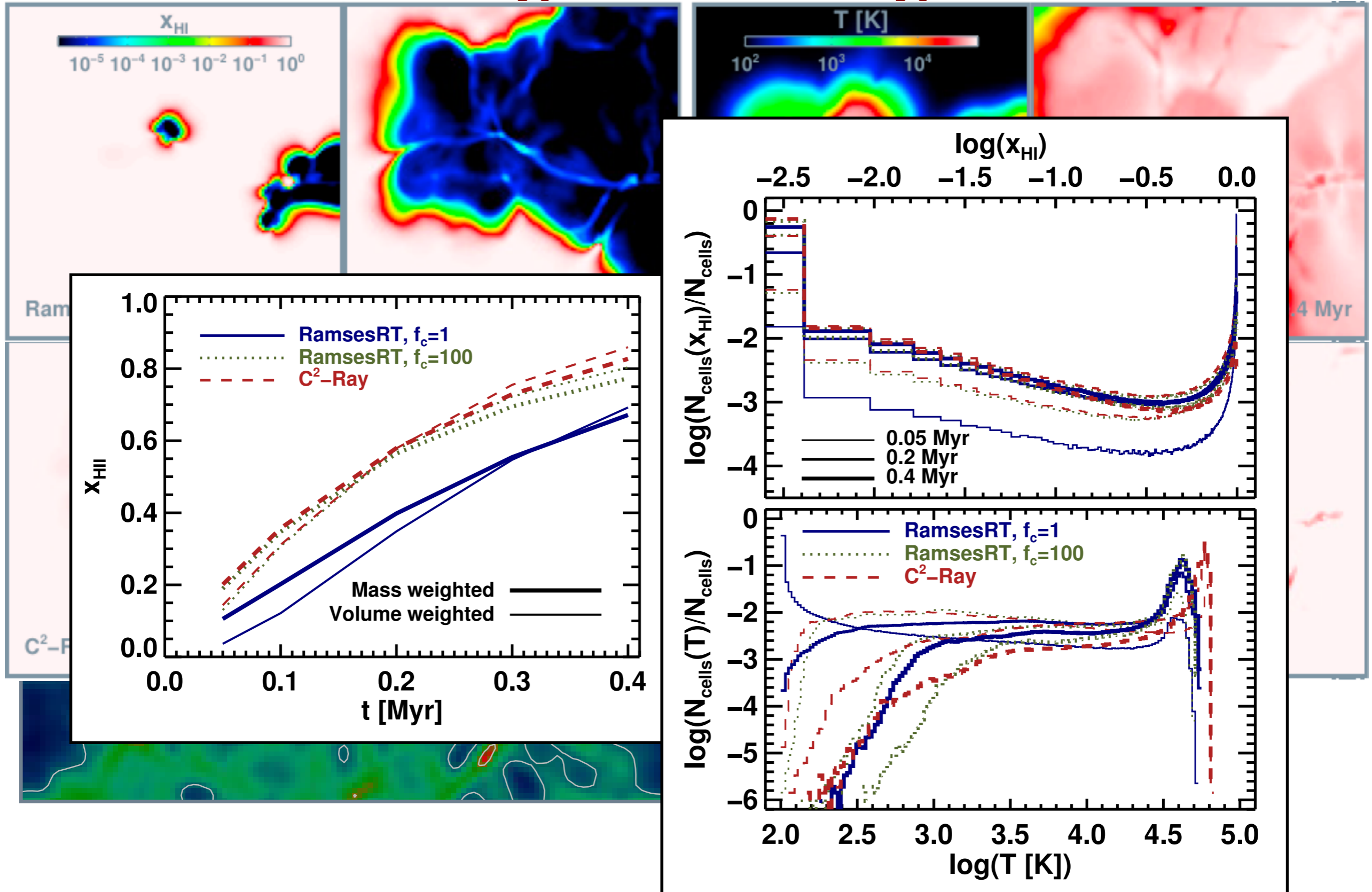
- For complete descriptions, see
  - Rosdahl et al. 2013: Standard Iliev tests
  - Rosdahl & Teyssier 2015: Radiation pressure and diffusion
  - Bisbas et al. 2015: Starbench code comparison project



# Iliev 2: HII region expansion



# Iliev 4: Ionizing a cosmological volume



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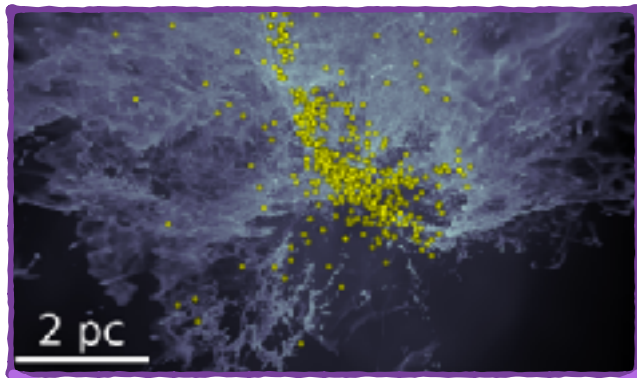
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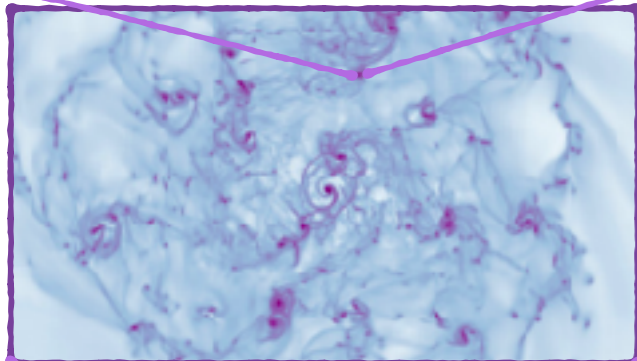
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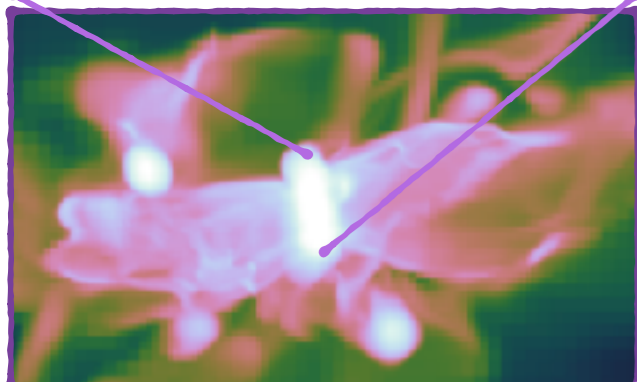
# Science with RAMSES-RT



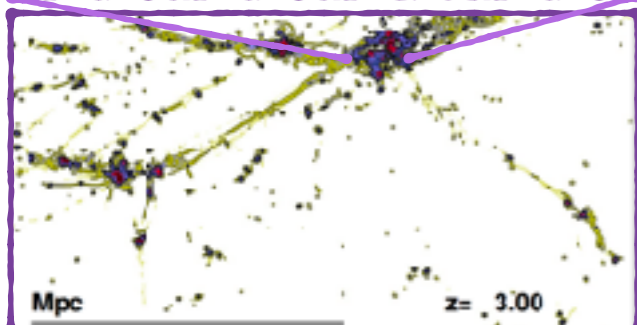
- **Molecular clouds**, sub-pc to pc scales  
Gavagnin, Bleuler, Rosdahl, & Teyssier (2017)  
Kimm, Cen, Rosdahl, & Yi (2016)  
Geen, Hennebelle, Tremblin, & Rosdahl (2015, 2016)  
Geen, Rosdahl, Blaizot, Devriendt, & Slyz (2015)  
Geen, Soler, & Hennebelle (2017)



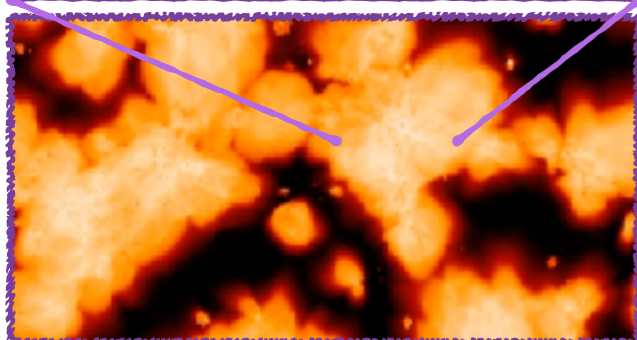
- **Galaxy-scale stellar feedback**, pc to kpc scales  
Butler, Tan, Teyssier, Rosdahl, van Loo, & Nickerson (2017)  
Rosdahl, Schaye, Teyssier, & Agertz (2015)  
Rosdahl & Teyssier (2015)



- **Feedback from active galactic nuclei**, pc to kpc scales  
Costa, Rosdahl, Sijacki, & Haehnelt (2017a,b)  
Roos, Bournaud, Renaud, Gabor, Dubois, Rosdahl, Perret, & Teyssier (2017)  
Bieri, Dubois, Rosdahl, Wagner, Silk, & Mamon (2016)



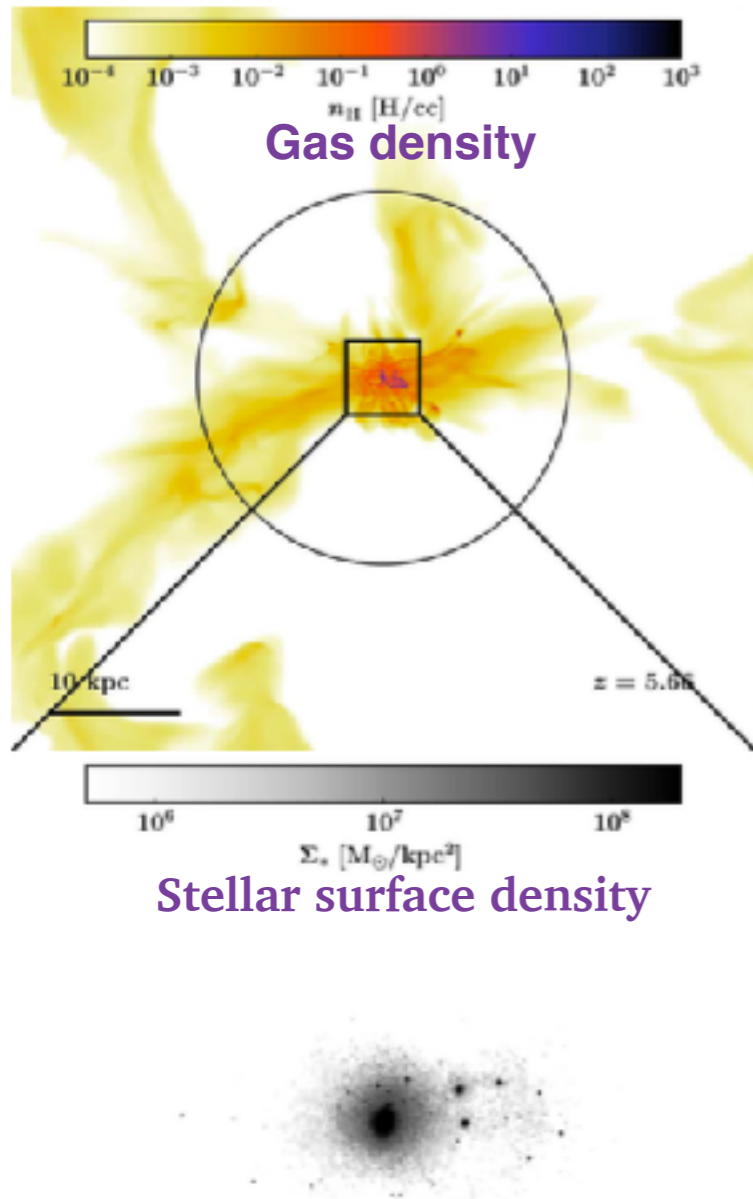
- **Properties of the circum-galactic medium**, kpc to Mpc scales  
Rosdahl & Blaizot (2012)  
Trebitsch, Verhamme, Blaizot, & Rosdahl (2016)



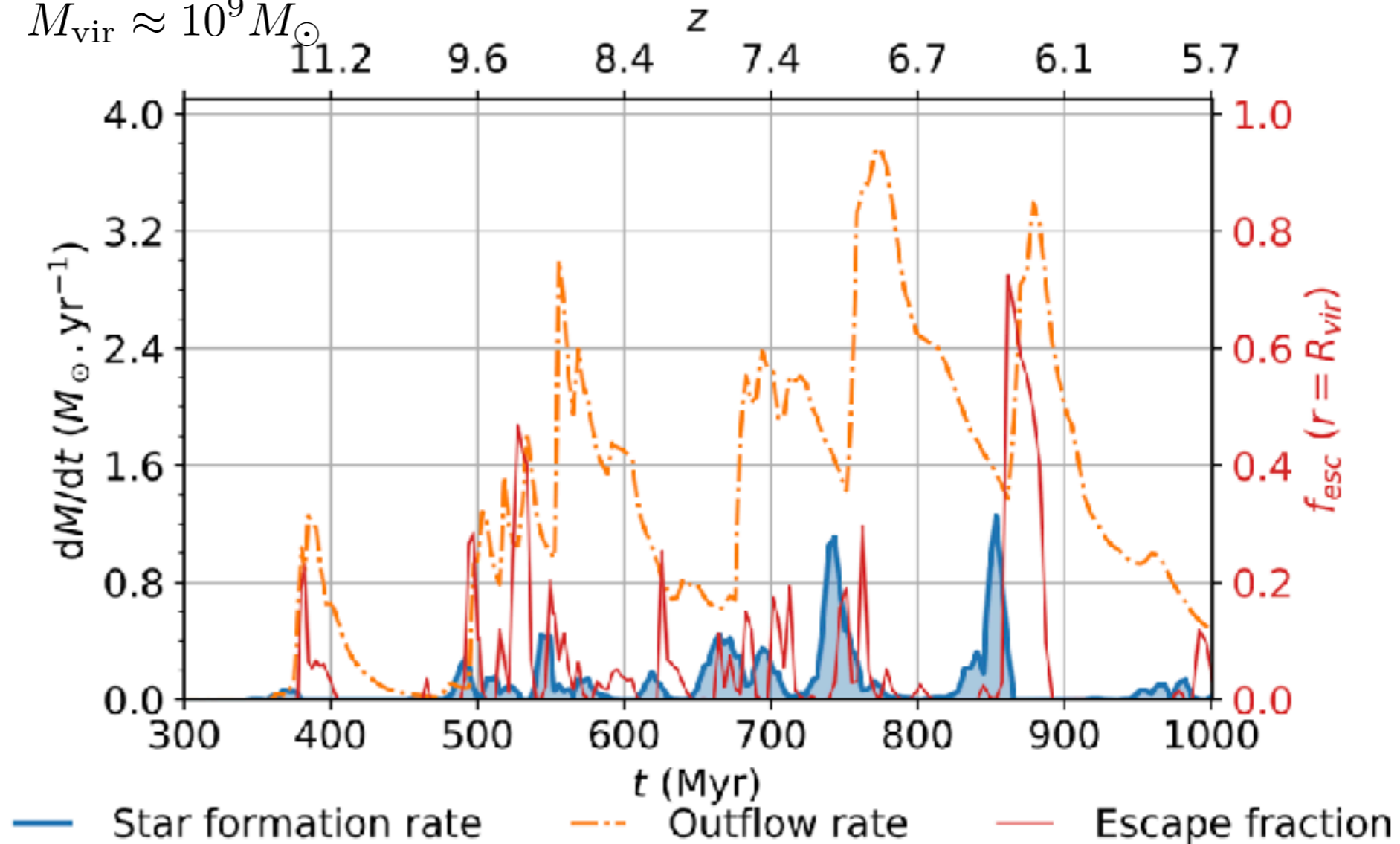
- **Reionisation and escape of ionising photons**, pc to Mpc scales  
Kimm & Cen (2014)  
Trebitsch, Blaizot, Rosdahl, Devriendt, & Slyz (2017)  
Kimm, Katz, Haehnelt, Rosdahl, Devriendt, & Slyz (2017)  
Katz, Kimm, Sijacki, Haehnelt (2017)

# UV escape from galaxies during reionisation

Trebitsch, Blaizot, Rosdahl, Devriendt, & Slyz (2017)



$$M_{\text{vir}} \approx 10^9 M_{\odot}$$



1 kpc

$z=5.68$

Low average escape fractions,  
regulated by SN explosions

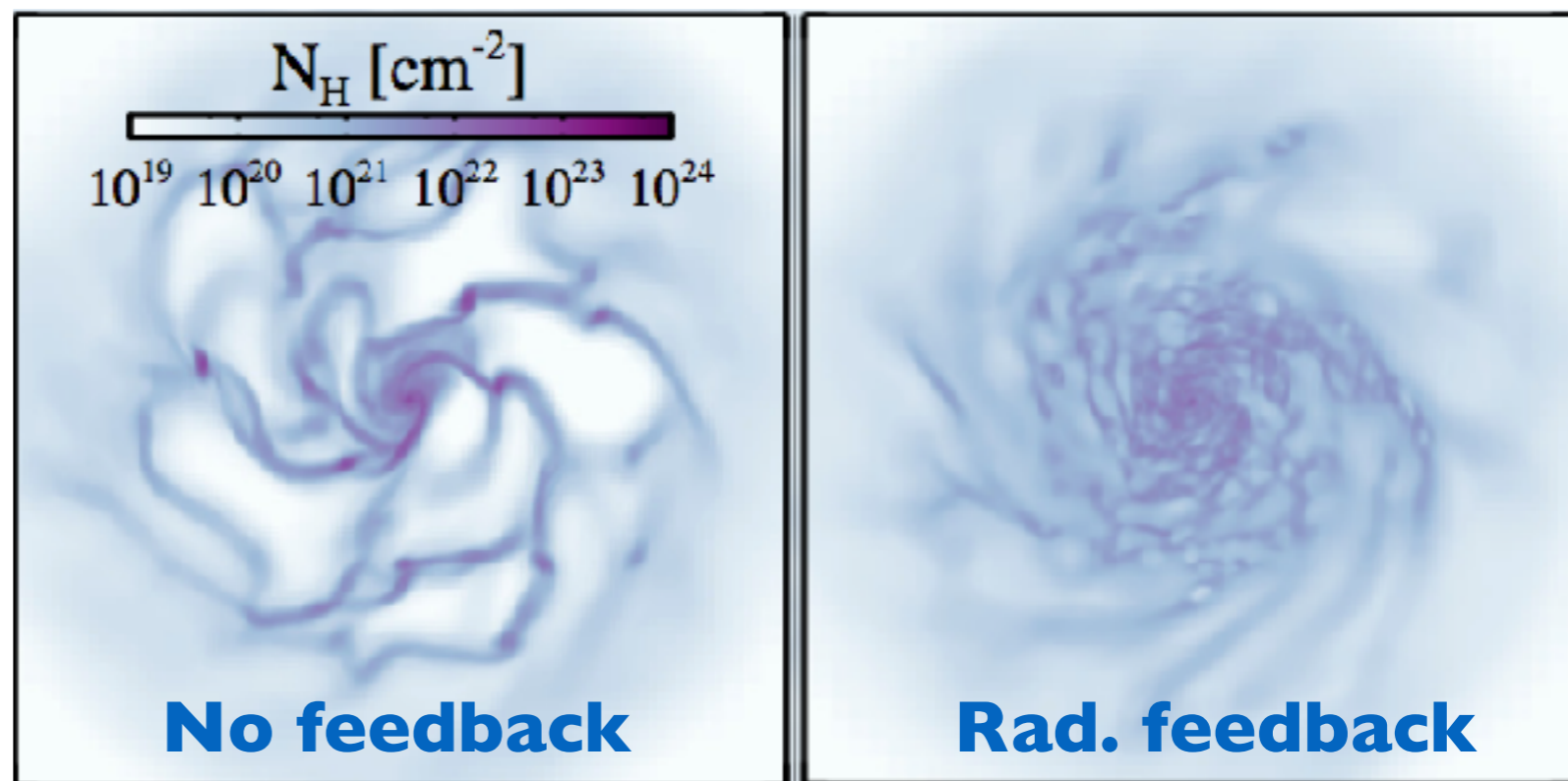
# Radiation feedback in isolated disk galaxies

Rosdahl et al. 2015

What is the role of stellar radiation feedback in galaxy evolution?

## Results:

- Considerable effect in low-mass galaxies.
- Smoother gas distribution and reduced star formation.
- Photoionisation heating dominates over radiation pressure (optically thin disks).
- More coming soon for high-redshift ULIRGs



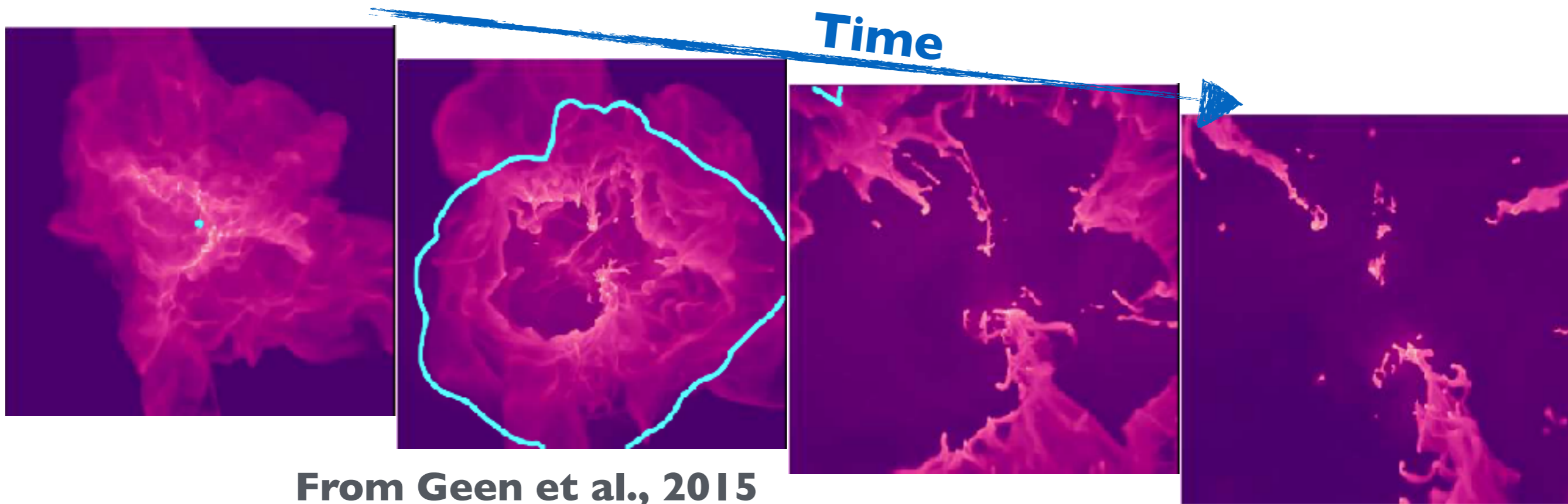
From Rosdahl et al. (2015)

# Photoionisation feedback in a stellar nursery

Geen et al., 2013, 2015, 2016, 2017

Gavagnin et al., 2017

**Studies of the photo-evaporation of star-forming clouds with sub-pc resolution: SN momentum boost, SF regulation and effects on the new-born stellar cluster dynamics**

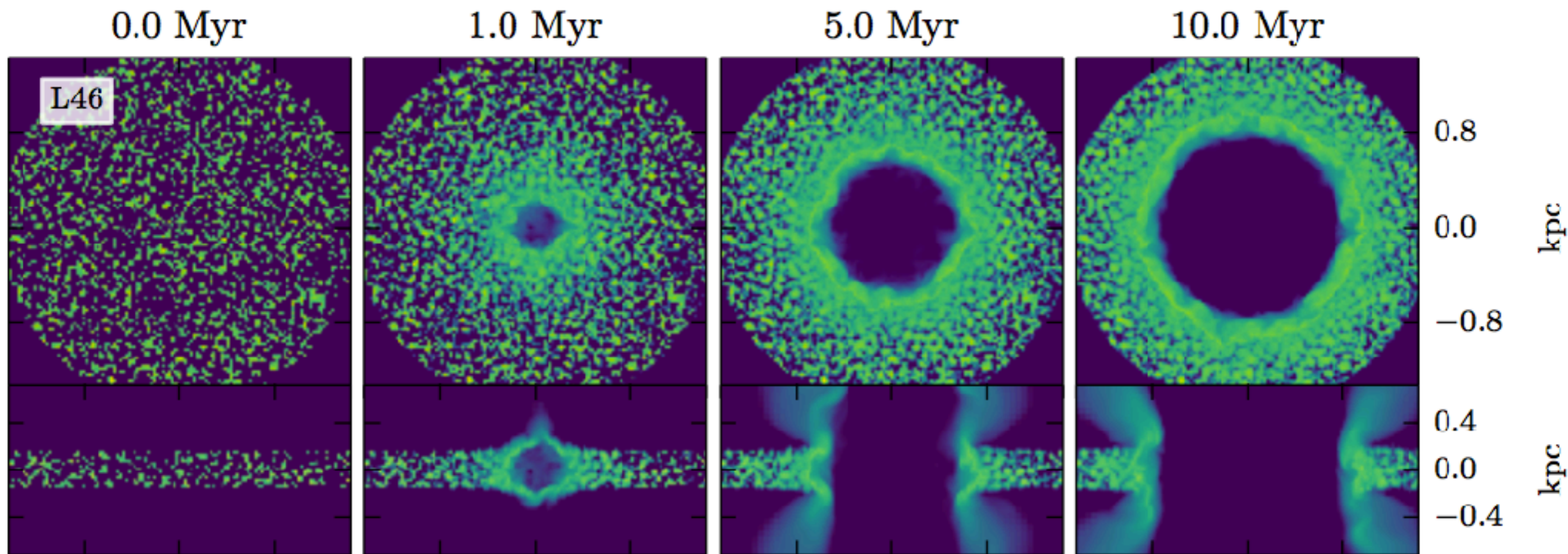


# Outflows driven by quasars via radiation pressure

Bieri et al. 2016

How efficiently can  
multiscattering IR radiation  
generate AGN outflows?

$$\dot{p}_{\text{rad}} = \frac{L_{\text{Opt}}}{c} \tau_{\text{IR}} \quad ???$$



The radiation can push out fast winds, but the boost is much weaker than tau.

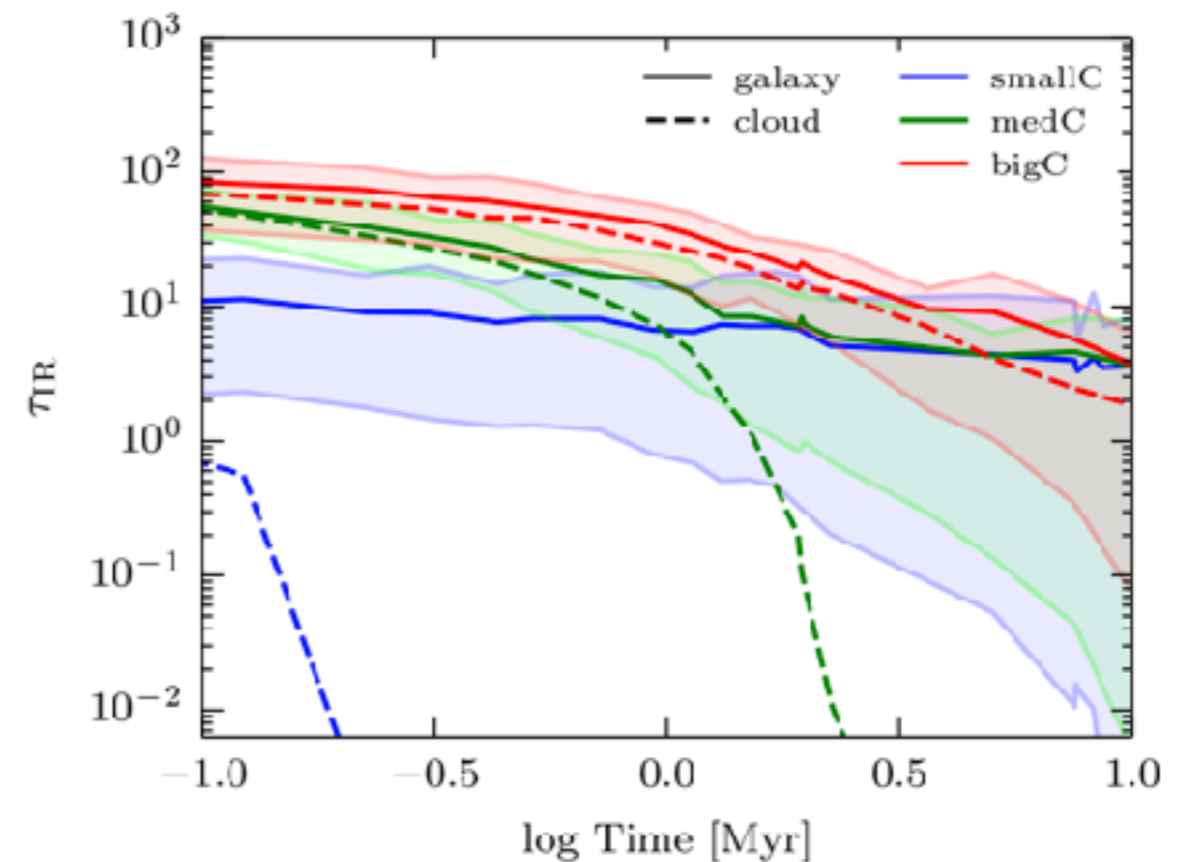
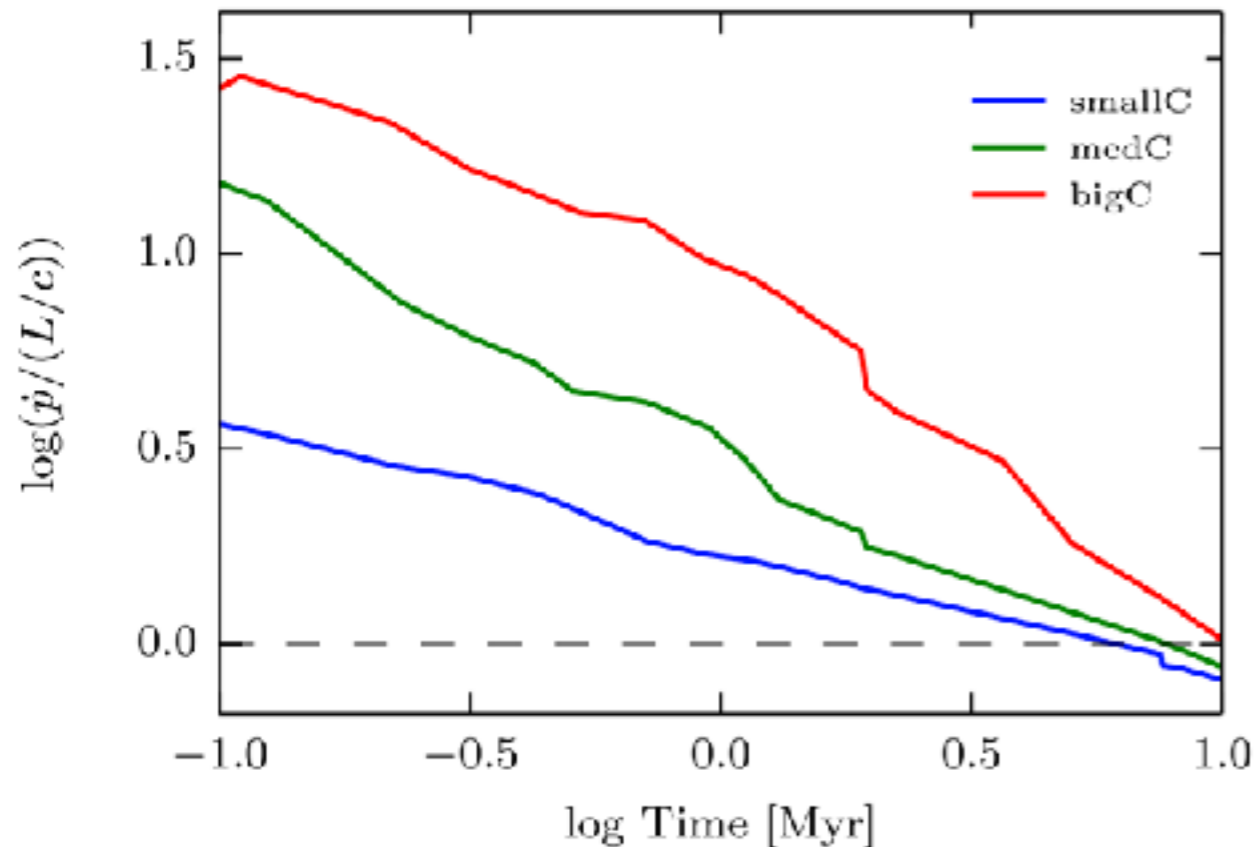


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# Future developments in Ramses-RT

- Speedup:
  - Implicit RT solver to get rid of reduced light speed?
    - Not clear if there is always an advantage...the system should preferably be 'slowly' evolving
- Coupling radiation with metal cooling
- H<sub>2</sub> chemistry - almost there
- Dust model, e.g. production, growth, destruction  
...the physics is complex and not well known

# Summary

## What is RAMSES-RT?

- Publicly available RHD extension of RAMSES
- On-the-fly radiation emission, transport, and absorption of H, He, and dust, using the M1 moment method
- Photoionisation, radiation pressure and dust scattering, correct in free-streaming *and* diffusion limits

## Why?

- To predict observable properties of gas
- To study radiation feedback on (sub-) galactic scales
- To study reionisation and escape fractions